# The Mathematics, Physics, and Computation of Music

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**Abstract**

Our project explored using musical ideas and concepts to assist in the teaching of physics and computer science. After some months of planning and research, we implemented our findings in Physics 1200, a course taught by William Pezzaglia, a member of our team. We developed an interactive computer simulation of vibrations along a string, and a course module where students build a wind instrument, the didgeridoo of aboriginal Australia. Challenges faced by the students presented opportunities for strengthening their grasp of STEM topics and supporting critical thinking skills as well as information literacy. These challenges evoked imaginative approaches by the team of faculty members to curriculum development for student-centered active learning and inquiry based learning.

**Student Learning and Success Focus**

Students at CSU East Bay often have difficulty learning mathematical and scientific subjects, as shown by the statistic that in Fall 2011 73% of CSU East Bay entering freshmen needed to take at least one remedial mathematics class (Krupnick 2011). In our project, we attempted to help improve competency in STEM subjects by using music, an art form which has clear mathematical (through music theory and the very way music is conceptualized) and scientific (primarily through acoustics) corollaries, as an entry point.

We began our project with a rather ambitious intention of using music as a teaching tool in existing and potential computer science, physics, and library, courses with the goal lf also developing ways incorporate STEM subjects into music courses where it was possible. During Summer 2012 and continuing into the fall quarter of that year, we held regular planning meetings in order to decide exactly how to carry out what we had committed to in the grant proposal. In addition to our meetings, all faculty members continued to do individual research related to the project.

We ultimately decided to work on two related and complementary projects which were done in conjunction with each other. One involved providing materials and consultation for Physics 1200, “Behind the Music,” a course our physics faculty member teaches as part of the Beats, Physics, and the Mind cluster. The other aspect of our project involved developing a computer program that simulates the behavior of vibrating strings under certain conditions. In addition, we kept in mind possible uses of the material we developed for courses in the other disciplines we represent.

All of the team members have an interest in music. Two have degrees in music (one of whom also has a degree in library science, and functioned as our information literacy specialist), and the two others are accomplished musicians in addition to their qualifications in, respectively, physics and computer science. All team members are aware of and very interested in the various connections between music and mathematics and the sciences, and are committed to interdisciplinary approaches to education in STEM subjects as well as the arts.

One issue that arose at the beginning of the project was deciding how to implement our ideas and findings. There are several existing courses taught by team members where work of this kind would be useful. For example, one professor teaches a class in computer music synthesis, another teaches information literacy courses which include substantial numbers of music, mathematics, and computer science majors, and a third teaches music theory courses where elements of mathematics and acoustics are relevant. Although we hope that all of these will be places we can implement our work in the future, we determined that the Physics 1200 course would be a good test course. This course is offered in the Spring Quarter, was therefore the first course on the class schedule in which we could implement our research, and was also a course where students in the past had expressed that they had some difficulties with the material.

Physics 1200 is part of the "Beats, Physics, and the Mind" freshman cluster, and students in it study musical acoustics. Before taking the physics course, they have already taken a Music Department course on audio production as part of the cluster (the third course in this cluster is a Philosophy course). Students already, therefore, have an interest in and some knowledge of music before taking Physics 1200. At the same time, they have varying amounts of musical training, so no particular amount of musical knowledge can be assumed. These students had already taken the required Information Literacy course, and our work on this grant provided opportunity to reinforce their learning of those concepts and skills as well.

The team members from physics and computer science created a computer program that simulates a vibrating string, and allows the user to perform experiments similar to those performed with an actual vibrating string apparatus used in physics lab experiments. Based on a commonly-used physics laboratory apparatus for studying vibrating strings (Figure 1), the software provides a more intuitive control interface, and generates audio in real time from the simulation of the vibrating string (Figure 2). The software, executables for MacOS, Windows, and Linux, as well as its source code have been posted to the web, and has been added to the CSU's MERLOT open courseware catalog.

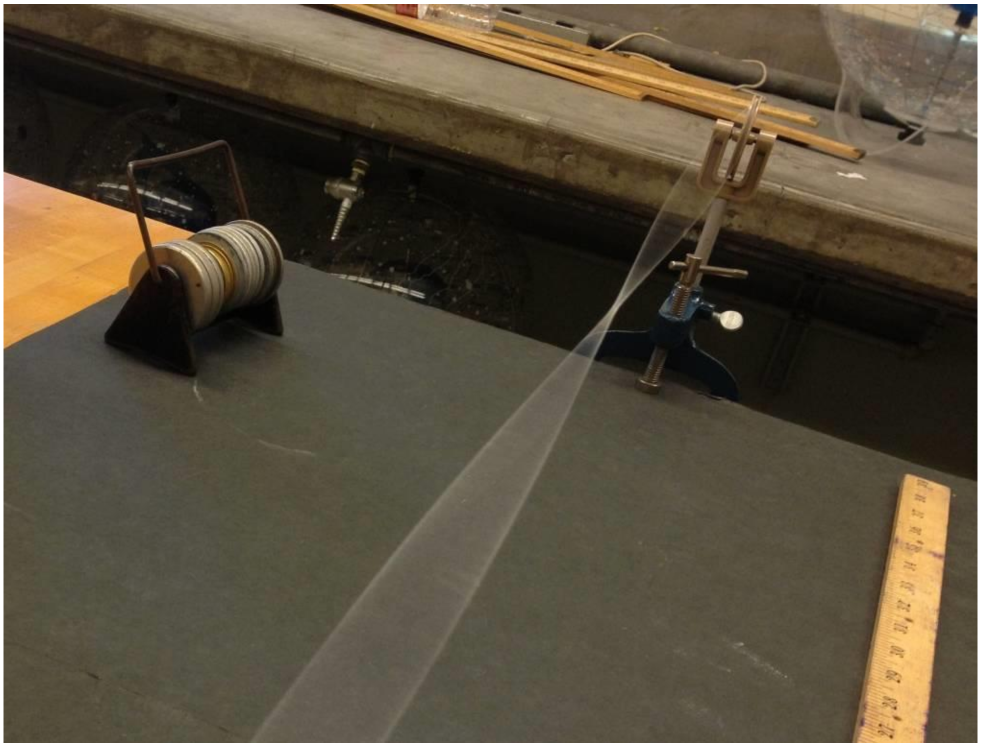

Figure 1. The physics laboratory apparatus

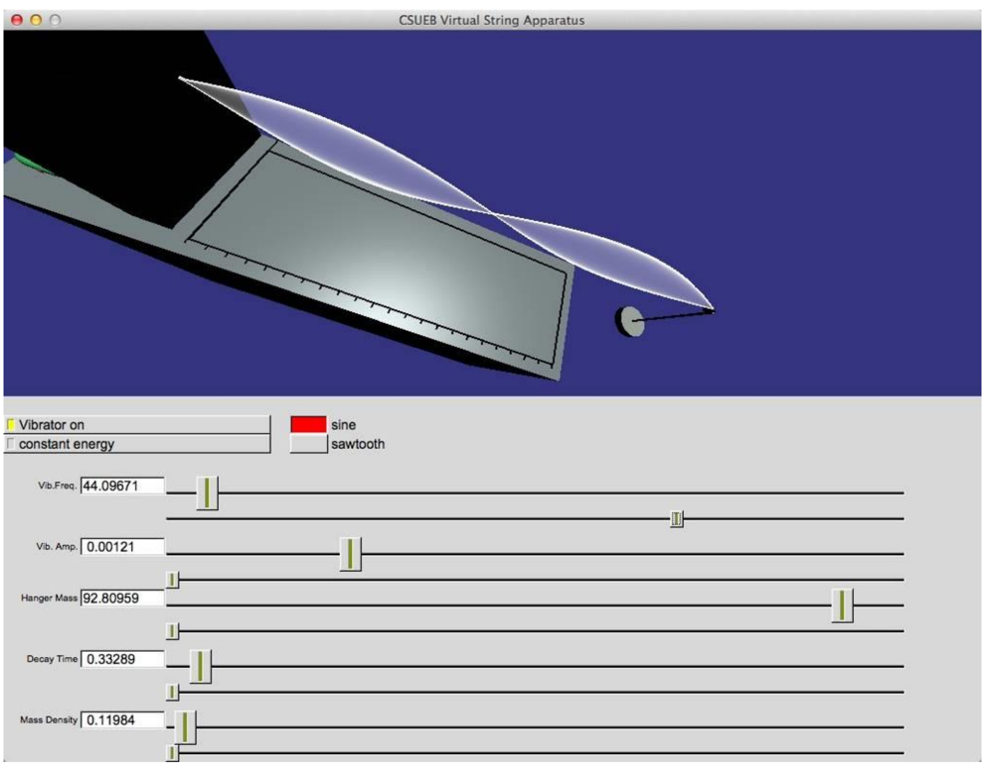


Figure 2. The string simulator

The program allows students to interact with a vibrating string by controlling the speed of a (simulated) mechanical vibrator attached to one end of the string. The vibrations created in the string travel along it, and are reflected back from the attachment point at the other end. With the right set of parameters, various phenomena can be seen. For example, the speed of the wave along the string can be changed by setting various parameters, and slowed down to the point where the motion is obvious. Most usefully, setting the vibrator frequency to create resonances or "standing waves" guides students to discover "harmonics" (at multiples of a base frequency).

The source code to the vibrating string simulator is available (at https://github.com/wthibault/MoM ), but most users would prefer to download an executable (at http://www.mcs.csueastbay.edu/~tebo/StringApparatus/ ) for Mac, Windows, or Linux. The source code leverages several open source libraries, and its study by upper-division Computer Science majors can improve their skills in the discipline.

The other significant aspect of our project was development of an instrument-building module for Physics 1200. We used as our demonstration instrument the Australian aboriginal instrument the didgeridoo. This was useful for Physics 1200 for several reasons. The first is that the didgeridoo is a relatively simple instrument, and is therefore easy to construct. Traditionally fashioned from eucalyptus branches hollowed out by termites, a very satisfactory didgeridoo that sounds very close of an "authentic" one can be built from PVC pipe purchased from a home-supply store.

There were a number of things that students were able to learn from building and playing these instruments. Working in teams of four or five, the students were assigned to construct, decorate with design, test acoustic properties, compose for, and perform on their instruments. In a five-person team each student was responsible for one of these areas, but all participated in planning and discussion (on four-member teams, either one non-essential area was dropped, i.e. “decoration/design” or two of the areas named above had to be done by one student or shared). Most germane to the discipline of the physics course, the physical and acoustic properties of the instrument were planned, measured, and compared to physical theory. Students did research into the use of the didgeridoo in Aboriginal cultures as well as into meaning of the way the instruments are traditionally and the meanings of the symbols used. The students also studied didgeridoo music and composed original music, some based on indigenous Australian music, some in newer styles using the instrument. At least one student in each group learned to perform the instrument. Students reported on their activities to the class, performed their compositions, and wrote up their results. Besides the course’s instructor from the physics department, the team members from the music department and the library lectured to the class, served as resources for the students, and evaluated the results.

A majority of the projects had interesting and successful outcomes, with students learning a variety of things. A primary goal was for students to gain greater scientific competence, and almost all did the measurements of their instruments and evaluation of the acoustic characteristics successfully. Students also expanded their skills in creating music, something they had begun in their Audio Production class in the freshman cluster, and developed their information literacy skills by researching the acoustics, design, and music, and culture of the didgeridoo. Many of the projects reflected this, with creative and original approaches to instrument construction, designs that reflected Australian Aboriginal art in interesting ways (including some specific and meaningful symbols), and compositions that ranged from rather traditional pieces to songs that used the didgeridoo as an accompaniment to rapping.

Prof. Bickley provided guidance regarding CSU East Bay University Library research resources to the acoustics class as a whole and contacted the students responsible for that research component via email. The response was not as strong as hoped for, but the five students who responded (representing five of the ten teams) located appropriate material, and reflected that in their written reports. Strength of citations for the sources varied, though the information located clearly influenced the groups' decisions regarding decoration of the instruments. Several groups revealed understanding of the cultural role of the instrument as well. Resources that groups used included an excellent website from an Australian music (Didjshop.com), University Library databases *Art Full Text*, *Anthropology Plus*, *Oxford Music Online*, and, of course, the open web, with Google as the preferred search engine.

Dr. Miller from the Music Department helped with the musical aspects of the class's didgeridoo project, including demonstrating techniques of playing the instruments, assisting Dr. Pezzaglia in discussion of how various aspects of acoustic design (length, diameter, using a straight pipe as apposed to a curved or bent pipe) might affect the sound of the instrument, and providing guidance in how to compose for the instrument. He was also available to provide musical and compositional information, and although only two students contacted him independently, several students asked relevant questions before or during class meetings. In addition, Dr. Miller constructed a didgeridoo and discussed how to do this with the class to provide them with a model for their own work.

Our library and music faculty members also appeared as guest lecturers at a total of three class meetings. Their involvement helped to supplement Dr. Pezzaglia's teaching in Physics 1200 with particular information in information literacy and music in order to help the students do their projects more successfully. As successful as this was, we feel that an even greater emphasis on team teaching done earlier in the quarter could be even more useful. Prof. Bickley and Dr. Miller also attended the presentations the students gave of their projects, and assisted Dr. Pezzaglia in evaluating the projects and assigning grades to them.

In addition, several students from the Music Department's New Music Ensemble, which is directed by our music faculty team member, met with the Physics 1200 class at two class meetings to discuss their own instruments. They discussed and demonstrated a variety of instruments, including percussive instruments at one meeting (drums, xylophone, bells, etc.) and at another string and wind instruments (trumpet, flute, clarinet, guitar, violin, accordion). The discussion included instrumental capabilities, acoustical properties, and the acoustical reasons that various instruments sound the way they do. Because of scheduling issues between the new music ensemble and the physics class, the demonstrations occurred late in the quarter; it is possible that arranging for these earlier would have been even more useful.

The learning outcomes of this project relate to several of Cal State East Bay's Institutional Learning Outcomes. Students in the class were able to learn about the connections between music and physics through the study of the acoustics of the didgeridoo, and had to apply analytical thinking and quantitative reasoning to successfully design and construct their instruments. In order to do this, they had to apply critical and creative thinking to their projects.

Working in teams required the students to work collaboratively in an effective manner, and to communicate their ideas and listen to other members of their teams in order to achieve success. Students' research into the cultural aspects of the didgeridoo enabled them to expand their knowledge of other cultures and to make intercultural connections. And finally, in order for projects to be successful, the students had to demonstrate knowledge of a specialized area of musical acoustics use that knowledge in practice, and draw conclusions from what was learned.

**Background**

The available scholarship in music as it relates to mathematics, physics, and computer science seems to be of two principal types. One type is rather practical, and consists of articles in music-education journals that teach students how to build simple string and wind instruments as well as descriptions of how to construct instruments on hobbyist and culturally-oriented web sites (we referred to one of these, didgshop.com, rather extensively in our work in teaching students how to build and play didgeridoos).

Another type of scholarship takes the form of books and articles written by specialists in the various disciplines we represented. Many of these books and articles are quite good, but many do not completely achieve a real interdisciplinary synthesis (nor do they need to, depending upon their scholarly objectives). There is also a developing body of work that involves interdisciplinary collaboration, but the nature of these requires the reader to have substantial knowledge of, for example, music and mathematics or of music and physics. There are some good sources written for general readers (Rothstein 2006). Many of the classic books on acoustics fall within this category, and are very useful for both physicists and musicians

In addition, there is some interesting material, mostly by musicians, that utilizes material from mathematics in a way accessible to musicians, and perhaps audiences, for example, the works of American composer Tom Johnson, which are frequently based on mathematical patterns that can be heard in performance. There is also a growing body of work that uses mathematics and physics as a component in music theory; these latter approaches are potentially very useful for advanced music courses.

The fields of computer science and physics have produced even more musically-oriented work. Computers have been used to produce and analyze musical sounds since the 1950s. At the present time, digital recording of music is ubiquitous and digital synthesizers and other digital means of producing music are extremely common. Our project used computer modeling of musical sounds from its inception, the first example of that being our recording and creating waveforms digitally for the September 2012 PEIL presentation. At the presentation, members of the team played live instruments while displaying the waveforms of those instruments. The point was to demonstrate how students could measure, quantize, and describe musical sounds using digital means, and learn the mathematics and physics of various sounds in this manner. The culmination of this in our project was the string simulator described above.

The string simulator source code illustrates the practice of posting code at the ubiquitous Github hosting service, which makes it amenable to modern programming teams of distributed workers students are likely to encounter in the real world. It also illustrates the modern programming approach of connecting collections of existing code into new applications.

**Research and Findings**

We learned several useful things from our research as well as from implementing our research in the Physics 1200 class. The team, in late summer 2012, began by doing rather general research in the mathematics of music, acoustics, computer modeling of musical and acoustical phenomena, and in the best ways to convey these ideas to students. Each of us initially surveyed the literature in our own disciplines pertaining to the project. The problems was, as an implementation grant, how to apply our findings; basic research was not the issue.

We found, over time, that an empirical approach was as important as doing basic research. All of us are experienced and successful teachers in our respective primary fields, but in an interdisciplinary project such as this, we had to figure out a method of incorporating all of our disciplines into one project. Focusing on one course, Physics 1200, which already has an interdisciplinary title, “Behind the Music,” gave us a way to develop and implement our ideas.

As mentioned above, we ascertained that the first course that was available for implementation was Physics 1200. We planned how to use our material in that course as well as discussing how our work could be applied to other courses, such as the required freshman library course that music majors take, Prof. Thibault’s computer music course, and a future course in the music department on musical acoustics.

One important conclusion we reached is that faculty teaching in an interdisciplinary manner such as we did in part of the physics course need to either be expert in all of the disciplines being covered, or need to work out how to communicate between disciplines in order to make the material work. It required faculty members from three departments—physics, music, and library—to make the didgeridoo project work and faculty members from two departments—computer science and physics—to develop the string simulator. Furthermore, all faculty members involved had to have at least an interest in and some knowledge of all of the other disciplines represented for the project to work as well as it did.

There does not seem to be a large amount of research about how to implement projects such as ours, and the large numbers of combinations of disciplines possible means that there might not be for many courses or interdisciplinary projects. We believe that our model could work for many sets of academic disciplines, but that preconditions for success include finding some common ground between the disciplines selected and for the faculty members involved so have knowledge of or a willingness to learn about the other fields involved. Our project worked because of the well-established mathematical and acoustical aspects of music and because the physics course we used as our demonstration project had as its subject musical acoustics. There are no doubt a large number of similar interdisciplinary projects that could be undertaken.

Given resources such as departmental willingness to offer appropriate courses and adequate release time or pay, we can envision using our project's approach in additional courses. To name just one discipline, there are many ways music courses ranging from the introductory to the advanced could use substantial material from acoustics, computer science, and mathematics that could give insight into music as well as help students learn more about the other disciplines. There are, no doubt, many other possibilities.

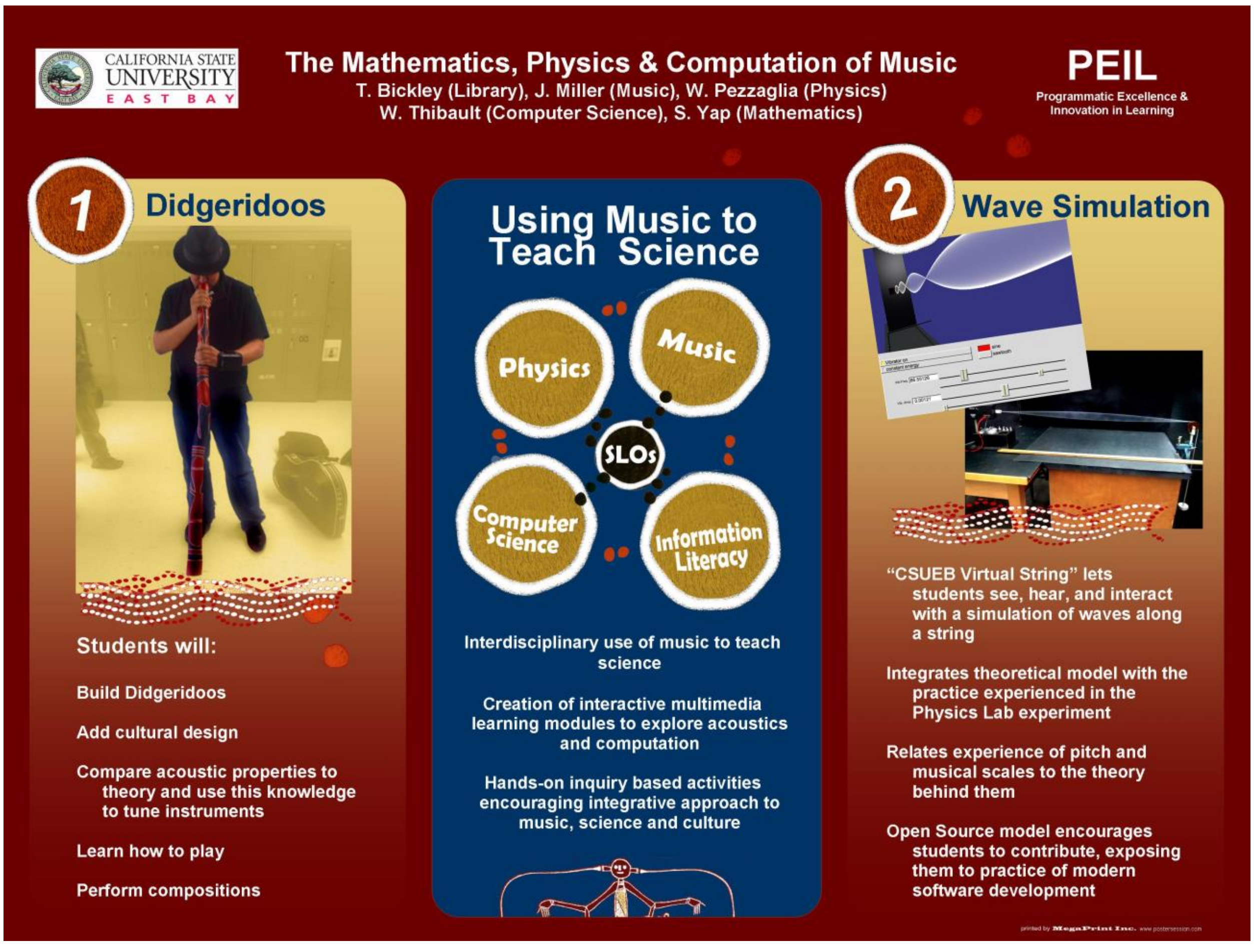


Figure 3. The poster from the spring 2013 PEIL poster session.

## Recommendations for Practice

We believe that what we discovered in the course of our project could be used in a variety of settings at CSUEB. The interdisciplinary nature of our work could be applied in numerous ways throughout the campus, although it seems clear that not all disciplines or combinations of disciplines could use this approach. Acoustics, although a branch of physics, has strong connections to music since it deals with, among other aspects of sound, musical sounds. Many of the methods used of measuring and analyzing sounds in acoustics can be done using traditional analog equipment or can be measured and quantified digitally. The latter means also means that acoustical phenomena can be synthesized digitally.

A sample example of how our method can be used makes this clear. Students could study how a trumpet, for example, works from several interrelated perspectives. They could study the mechanism of sound production, measure how the length and diameter of the trumpet affects pitch, and can investigate how the material the instrument is made of affects tone color. In addition, there are detailed aspects of instrument construction, such as how the valves work, that can be measured. Recordings of trumpets could be digitized and analyzed to determine the harmonic spectrum of the instrument; students could then attempt to synthesize a trumpet sound or create new trumpet-like instrument sounds digitally. And, students could research the history of the trumpet, learn the types of music played by the instrument, and learn something about the cultural origins and practice of the trumpet. Any or all of these aspects of the instrument could be explored to the depth appropriate for the particular course and the students. The approach for all of these areas could be project-centered, so that the students are discovering them through research and experimentation, not through lectures.

For our Physics 1200 project, we used an acoustically rather simple instrument, the didgeridoo, as our model. This was appropriate for a freshman cluster class in which all of the students were interested in music, but where the formal musical backgrounds of the students ranged from having no formal training to reading musical notation and being good performers on an instrument.

Depending upon the type of course and its prerequisites, the nature of student research and projects might vary, but the underlying principles would remain the same. For a music major class, for example, a high degree of expertise in music would be assumed, but only basic knowledge, and even some fear, of the mathematics and physics aspects of the course would be expected. In that setting, the music the students solve might be more sophisticated, but the mathematics and physics might be rather straightforward. Conversely, for an acoustics class involving physics majors, the physics problems might me rather complex, but the musical composition and performance aspects would be simpler than those required in the music course.

For the information literacy component, our library faculty team member observed promising evidence of student efforts at research. In the LEAP Value Rubric for Information Literacy (AAC&U 2013), he places most of their efforts between the benchmark (1) and the first Milestone (2). While seeking to embed the responsibility for accomplishing the research into the student teams, likely better results would have been forthcoming with additional meetings with the specific students working on that aspect of research. Ideally, this project could have been "foreshadowed" in the sections of LIBY 1210 (Introduction to Information Literacy) linked to the Beats, Physics and Mind cluster. Similarly, discussion in advance of this project in the cluster Philosophy course may have prepared these students for a richer experience and awareness of the cultural and aesthetic issues encountered with the didgeridoos.

We found the interdisciplinary interactions among diverse faculty to be invigorating and motivating, and encourage the University to continue to create opportunities for interdisciplinary collaboration. Faculty require release time to make these kinds of research efforts feasible, however.

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Tom Bickley (Library Faculty at CSUEB), teaches a required course (Introduction to Information Literacy) and serves as subject specialist for music, computer science, and philosophy. He co-edited the *Anthology of Essays on Deep Listening* (Deep Listening Publications, 2012), and is on the organizing committee for the 1st International Conference on Deep Listening: Art/Science at Renssalaer Polytechnic Institute. His musical life includes work as a composer/performer in experimental music. He holds a B. Mus. in music theory from the University of Houston, an M.A. in music from the American University, an M.Div. from Wesley Theological Seminary, an M.S.L.I.S. from the Catholic University of America (all in Washington, DC).

Jeffrey Miller is a Lecturer in the Department of Music at Cal State East Bay. He holds a B.M. from San Francisco State University and a Ph.D. in music composition from the City University of New York. He teaches music theory and composition, and has also directed the music department’s new music ensemble. In addition to his teaching activities, he is an accomplished composer whose works have been performed throughout the United States.

Physics Instructor William Pezzaglia regularly teaches courses in Astronomy, General Physics and Musical Acoustics at CSUEB. In 2007 he created the introductory course “Behind the Music”, teaching freshman about waves, sound and musical instruments. He holds a Ph.D. in theoretical physics from University of California, Davis. His research area is mathematical physics, with special interest in the nature of “spin” in electromagnetism and gravity. His other passion is ragtime piano. He often can be found either at a telescope or playing various instruments (tuba, trombone, clarinet) with various local bay-area folk singer-songwriters.

William C. Thibault received a B.S. in Computer Science from the University of New Orleans in 1981, and M.S. and Ph.D. degrees in Information and Computer Science from the Georgia Institute of Technology in 1985 and 1987. (Thesis title: "Application of Binary Space Partitioning Trees to Geometric Modeling and Ray-Tracing") As a Ph.D. candidate he worked at Bell Labs (now Lucent Technologies) in Murray Hill, NJ. Upon graduation, he took his current position on the faculty of the Department of Math and Computer Science at California State University, East Bay.


The grant team, from left to right: William Thibault, Bill Pezzaglia, Tom Bickley, Jeffrey Miller