The Microtones of Bharata's *Natyashastra*

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In about 200 CE, Bharata and Dattila penned closely related Sanskrit treatises on music in the *Natyashastra* and *Dattilam*, respectively (Ghosh 1951, lxv). Inside, they outline the essential intervals and scales of *Gandharva* music, a precursor to the Hindustani and Carnatic traditions. Their texts are the first known documents from south Asia that attempt to systematically describe a theory of intonation for musical instruments. At the core of their tonal structure is a series of twenty-two microtones, known as *sruti*-s.[1]

The precise musical definition of the twenty-two *sruti*-s has been a topic of debate over the ages. In part, this is because Bharata does not explain his system in acoustically verifiable terms, a hurdle that all interpretations of his text ultimately confront. Some authors, such as Nazir Ali Jairazbhoy and Emmie te Nijenhuis, suggest that Bharata believed the *sruti*-s were even (twenty-two tone equal temperament, or 22-TET), though they were not in practice (Jairazbhoy 1975, 44; Nijenhuis 1974, 14–16). Jairazbhoy concludes that it is impossible to determine the exact nature of certain intervals in Bharata's system, such as major thirds. Nijenhuis (1974, 16–19) takes a different approach, suggesting that the ancient scales were constructed using interval ratios 7:4 and 11:10, so as to closely approximate the neutral seconds and quarter tones produced by 22-TET. Prabhakar R. Bhandarkar (1912, 257-58) argues that Bharata's 22-TET system was approximate and still implied the use of 3:2 perfect fifths and 5:4 major thirds. P. Sambamurthy (1963) argues that the *sruti* system was based exclusively on cycles of 3:2 perfect fifths, modified to include 5:4 major thirds, but does not justify his position in the context of Bharata's descriptions. Cris Forster's (2010) interpretation of Bharata's text also includes 3:2 and 5:4 intervals, though he does not contrast his approach with other analyses constructed from simple interval ratios, like the one provided by Nijenhuis. Several other authors also use 3:2 and 5:4 intervals, but unlike Jairazbhoy, Nijenhuis, Bhandarkar, Sambamurthy, and Forster, they either place *sruti*-s above their respective notes rather than below, leading to fundamental differences in scale type, or do not consistently observe the *sruti* count between intervals described in the texts (Danielou 2010, 50; Kolinski 1961, 3–4; Fox Strangways 1935, 690–91).

I base my approach to the *Natyashastra,* a manual for the production of dance dramas, on the principle that Bharata's tonal theories are derived from musical practice in the context of ancient south Asia. Imagine that a *Gandharva* musician is sitting down to tune a stringed instrument. There are no computers, electronic tuners, *tanpura* boxes or "apps," no drone instruments of any sort, and no knowledge or practice of tuning with string length calculations (Forster 2010, 542–43; Widdess 1995, 7). Instead, their method was probably to

1. Throughout this text, pluralization of Hindi or Sanskrit words has been indicated with an "-s." *Sruti* (or *śruti*) is pronounced "shroo-tee."

tune by ear, relying on their perception of the musical intervals to provide the most recognizable reference points. With this in mind, my interpretation is founded on the criteria that Bharata and Dattila's tonal structures must have practically viable tuning procedures on the *vina*, the ancient seven-stringed instrument to which they were referring, while satisfying the theoretical descriptions and rules found in the *Natyashastra*.

To that end, this paper will examine the rationale behind defining the "consonant" interval (Ghosh 1961, 6–7), called *samvadi*, as a perfect fifth (3:2). The likelihood that a major third (5:4) was also used by musicians to tune their instruments will then be investigated. The implementation of these two intervals corresponds to five-limit just intonation, in which the first five partials of the harmonic series and their octave equivalents are the basis for tuning. Other theories of intonation, specifically 22-TET, quarter-tone, and three-limit (or Pythagorean) approaches, will be compared with five-limit just intonation in order to determine which most accurately fits the descriptions found within the *Natyashastra*. The various approaches will be assessed based on their practicality and the ease with which they can be tuned by ear, without relying on logarithmic mathematics or string-length calculations.

I hope that this analysis will help others to explore and expand upon Bharata's principles in their own music making, and to come to their own conclusions regarding the potential application of *sruti*-s to contemporary music. I've included many visual aids in order to illustrate the tuning sequences that may have been used in *Gandharva* music. I also offer a description of how my interpretation of Bharata's scales can be applied to both the ancient *vina* and the modern sitar. Though I often use the twenty-two *sruti*-s I describe herein on Hindustani instruments, they fall short of representing a definitive catalogue of notes. Instead, I see them as waypoints along a journey I continue every time I sit down to tune an instrument.

THE ANCIENT *VINA*

A stringed instrument called a *vina* was at the heart of Bharata's *sruti* system. Although he included chapters on other instruments, such as the human voice and the flute, the concept of *sruti* was initially described with reference to the *vina* in a chapter devoted to instrumental music and was excluded from his list of musical phenomena derived from the voice (Rangacharya 2010, 219, 244, 273, 297). Bharata described two types of *vina*-s: the seven-stringed *citra* to be played with the fingers, and the nine-stringed *vipanci* to be played with an ivory pick called the *kona* (Rangacharya 2010, 240). The string limit imposed by Bharata is an important clue that will provide insight into the likelihood of various tonal possibilities for the music of the *Natyashastra*.

Though the name *vina* has been attached to a variety of other instruments throughout history, many scholars now agree that Bharata was probably referring to an arched, bow-shaped harp. Because no physical specimens from Bharata's time have survived, this conclusion is based on descriptions of the *vina* in texts and the archaeological evidence provided by numerous temple reliefs and ancient coinage on which harps are depicted (Danielou 2010, 52; Forster 2010, 543–44; Williamson 2000, 11-16; Sambamurthy 1963; Kolinski 1961, 4; Coomaraswamy 1930; CoinIndia, n.d.). According to A.K. Coomaraswamy's (1930) analysis, this instrument sported seven or more strings (*tantra*), strung between a curved beam (*danda*) and a hollow, boat-like body (*bhanda*) covered in animal skin (*carma*). Several ancient representations of the harp-*vina* are shown in Figures 1, 2, and 3.

Figure 1. "Two seated female harpists, part of the chorus of a dance of apsarases, Bharhut, ca. 175 B.C. Indian Museum, Calcutta" (Coomaraswamy 1930, 253).

Figure 2. "Harpist (possibly Pañcaśikha) walking, accompanying a processional dancer, Amarāvatī, ca. 200 A.D. British Museum" (Coomaraswamy 1930, 253).

Figure 3. "Samudragupta, gold dinar, c. 335–375 CE. King seated left on a couch, playing the *vina*." Used by permission of CoinIndia.

In central India, a harp tradition still survives with the Gogia Pardhan people, who play a little-known five-stringed bow harp called the *bin-baja*. This instrument may have descended from the harp-*vina*. Its construction, which can be seen in Figure 4, corresponds to the ancient depictions (Knight 1985). The *waj* is another rare harp from the Afghan province of Nuristan with potential origins in *Gandharva* music, though its design is somewhat different (Irgens-Møller 2009; Knight 2007, 1985). Historical evidence also supports the opinion that the *vina* migrated to Burma around the year 500 CE and is an ancestor of the modern Burmese *saung-gauk* ("Burmese harp") as well as the harps played by the Karen people of Myanmar and

Figure 4. Ram Prasad Pandro of Kokomata village playing the *bin-baja* (1982). Courtesy of Roderic Knight.

Thailand (Kersalé 2016; Williamson 2010; Becker 1967). Recently, Patrick Kersalé and a number of Cambodian craftspeople recreated some of the ancient instruments of Cambodia, modeling the previously extinct *pin* off of the harp-*vina* from which it is believed to have descended (Kersalé 2016).

The ancient harp-*vina* should not be confused with contemporary instruments bearing similar names, such as the *rudra vina* of North India, which is a fretted stick zither, or the South Indian *vina,* a fretted lute. While there is evidence that instruments of the lute and stick-zither types existed in ancient India, the ubiquitous depictions of harps and the description in Bharata's and other texts suggest that the *Natyashastra* was probably referring to the latter. For example, Coomaraswamy (1930, 252–53) describes a five-stringed lute, similar in appearance to the Japanese *biwa*, which also appeared in depictions alongside the harp-*vina*. However, observing that "we have a very large number of representations, ranging backwards from the late Gupta period to the beginning of the second century BC . . . in which the vina is consistently depicted as a kind of harp" (244), Coomaraswamy concludes that "the descriptions and actual representations are so consistent and so much in agreement that we are justified in speaking of the harp-vīṇā here described as *the* old Indian vīṇā" (245). Alastair Dick (2001) also mentions the existence of other ancient stick-zithers and lutes, but writes that at the time of the *vina*'s first documented occurrence in writing (in the *Yajurveda,* ca. 1000 BCE) "it is clear that *vīṇā*, unqualified, at this period denoted the harps or bow harps." According to Dick (2001), its role as an instrument for court entertainment was "confirmed for the harp by the earliest (mainly Buddhist) art from the 2nd century BCE until about the 6th century CE." This corresponds to the era in which the *Natyashastra* was probably written (Ghosh 1951, lxv).

With regard to the structural considerations of the ancient harp-*vina*, instrument builder and tuning analyst Cris Forster (2010, 543) makes the following additional observations:

1. There is no evidence for the use of tuning pegs;
2. The strings were probably tuned using tuning cords, which were subject to "creep and slippage";
3. There was evidently no post to provide support between the open ends of the bow-shaped design.

These details are confirmed in the contemporary Indian bow-harp (*bin-baja*) as described by Roderic Knight (1985). The tuning difficulties inherent in its design along with the seven- and nine-string limits imposed by Bharata's description (Rangacharya 2010, 240) can both serve as crucial limiting factors when analyzing the tonal content of the *Natyashastra.* These considerations suggest that complex tuning procedures, with steps exceeding the number of available strings, were less likely than simpler possibilities. Forster (2010, 543–44) concluded that "it is extremely unlikely that the ancient harp-*vina* was used to tune technically difficult scales," and that, "the harp-*vina* was probably tuned to 'octaves,' 'fifths,' 'fourths,' and 'thirds.'"

WHAT BHARATA WROTE

Each of Bharata's musical rules must be understood before conclusions can be drawn about the application of twenty-two *sruti*-s on the harp-*vina*. The list below summarizes his essential principles of intonation.

1. "The *svara*-s are seven: *Sadja, Rsabha, Gandhara, Madhyama, Pancama, Dhaivata,* and *Nisada*" (Rangacharya 2010, 219). Corresponding to the seven primary pitch positions in both *Gandharva* and modern Hindustani and Carnatic music, the *svara*-s are represented throughout this paper as Sa, Ri, Ga, Ma, Pa, Dha, Ni, in ascending order.[2] The contemporary system, called *sargam*, can be roughly correlated with Western movable-do solfège, as shown in Figure 5.[3] A shorthand notation, outlined in Figure 6, will also be used to list the chromatic scale of twelve consecutive half-steps as follows: S r R g G m M P d D n N S.

2. The seven *svara*-s are further subdivided into twenty-two microtonal positions called *sruti*-s. The number of *sruti*-s located between each pair of notes determines the precise qualities and pitch positions of the *svara*-s. Though opinions differ regarding their potential application in contemporary Hindustani and Carnatic music, it is explicitly stated in Bharata's text that the ancient tonal system comprised twenty-two *sruti*-s.

3. *Vadi* is the first of four special classifications used by Bharata to describe pitch relationships. The precise definition of *vadi* is somewhat ambiguous, though it could be interpreted as either a prominent note, a single note in and of itself, or a note that serves as the reference point for subsequent intervallic comparisons (Rangacharya 2010, 219; Nijenhuis 1970, 19).

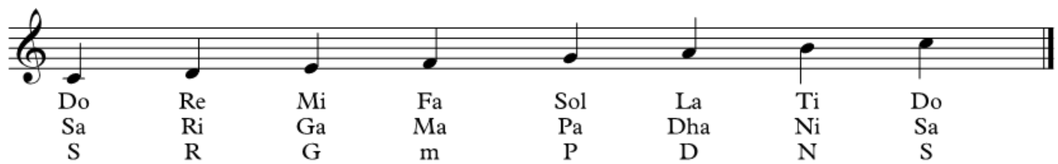


Figure 5. Correspondence between *sargam* syllables and movable-do solfège.

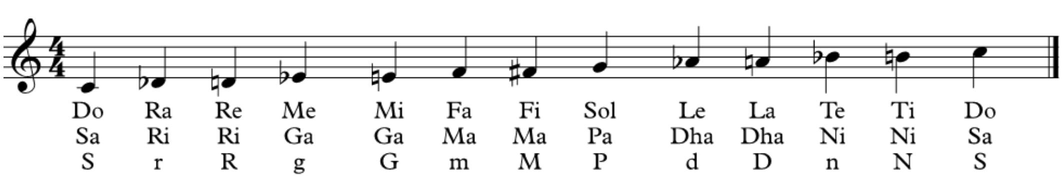


Figure 6. *Sargam* shorthand for chromatic pitches.

2. "Sadja is [the starting point] in the sadjagrama. From this one the third [sruti] upwards is, no doubt, rsabha. From this one the second [sruti] is gandhara, from this one the fourth [sruti] is madhyama" (Nijenhuis 1970, 19).
3. In *sargam*, the second degree may be written as "Ri" or "Re." I have chosen to use "Ri," in part to distinguish it from the use of "Re" in Western solfège.

4. *Samvadi* is the second of Bharata's special classifications, and is the defining interval of his system. It indicates that two *svara*-s are separated by either nine or thirteen *sruti*-s, and "harmonise with each other" or sound "consonant" (Rangacharya 2010, 220; Ghosh 1961, 6-7).[4] In a twenty-two *sruti* octave, nine- and thirteen-*sruti* intervals are inversions of each other. Bharata initially lists the following examples: Sa-Ma, Sa-Pa, Ri-Dha, and Ga-Ni (Rangacharya 2010, 219).[5]

5. *Vivadi* is the third classification, indicating that two *svara*-s are separated by an interval of either two or twenty *sruti*-s which, like the *samvadi* intervals, are inversions of each other. Bharata lists Ri-Ga and Dha-Ni as examples (Rangacharya 2010, 219).

6. *Anuvadi* is the fourth classification, and is assigned to any *svara* that is not *vadi, samvadi,* or *vivadi*.

7. There are two tonal systems, or *grama*-s (literally "villages"; Macdonell [1893] 2006, 88), known as *shadja* (or *ṣaḍja*, pronounced "shə-jə") and *madhyama,* each containing seven notes. In the *shadja grama*, Sa and Pa are *samvadi*, and Ri and Pa are not. The *madhyama grama* is identical, except that Pa and Ri become *samvadi* while Sa and Pa are not. This is another way of saying that Pa is one *sruti* lower in the *madhyama grama*. Bharata's comparison of *samvadi* pairs in order to distinguish between the *grama*-s suggests that the *samvadi* interval could have been used as a basis for tuning.

8. Bharata assigns *sruti*-s to the *svara*-s of the *shadja grama* as follows: Sa (4), Ri (3), Ga (2), Ma (4), Pa (4), Dha (3), and Ni (2) (Rangacharya 2010, 220). Figure 7 depicts the *shadja grama* in such a way that each dash or note name indicates a *sruti* position.

9. The *madhyama grama* is similar to the *shadja grama*, except that Pa is reduced by one *sruti*: Ma (4), Pa (3), Dha (4), Ni (2), Sa (4), Ri (3), and Ga (2) (Rangacharya 2010, 220). Two methods for modulating between the *grama*-s are described in the *Natyashastra*. These produce two possible layouts for the *madhyama grama* that share the same *sruti* count. One places Ma on the first string, while the other places Sa on the first string, as shown in Figure 8.

In both *grama*-s the *sruti*-s were placed below, not above, the *svara*-s they were attached to. In other words, the *sruti*-s combine to form an interval between their respective *svara* and

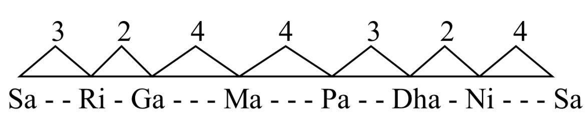
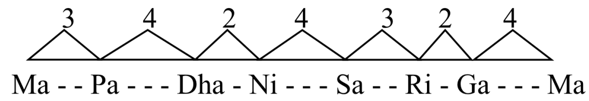


Figure 7. *Shadja grama.*

4. Bharata's definition of *vadi* and *samvadi* differs somewhat from the modern perspective, in which the *vadi* is seen as the most prominent note in a melodic form, and the *samvadi* as the second most prominent.
5. Nijenhuis's (1974, 16) interpretation was partially based on her position that Ma and Ni were not *samvadi* in the ancient system because of their exclusion from Bharata's list. However, Ma and Ni are separated by nine *sruti*-s according to Bharata's allocation of *sruti*-s to the various *svara*-s, making them *samvadi* by definition.



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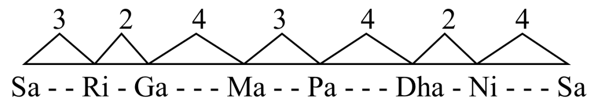

Figure 8. *Madhyama grama.*

the note below it. For example, when Bharata wrote that Sa has four *sruti*-s he meant that the interval between Sa and the Ni beneath it is a four-*sruti* interval.

Some authors, such as Kolinski (1961, 3–4), Hornbostel (quoted in Kolinski 1961, 4) and Fox Strangways (1935, 690–91), take the position that Bharata's *sruti* intervals were placed above the *svara*-s they were attached to (for a critique of other writers who took the same position, see Bhandarkar 1912, 187). This approach conflicts with the text of the *Natyashastra* in two ways. First, this method widens the gap between Ri/Ga and Dha/Ni to three *sruti*-s in both *grama*-s (see Figures 9 and 10), contradicting Bharata's statement that the two-*sruti vivadi* interval should be found in those positions (Rangacharya 2010, 219). Second, it incorporates a non-*samvadi* interval of eight *sruti*-s between Ri and Pa in the *madhyama grama,* as shown in Figure 10. This was expressly prohibited by Bharata: "There is a traditional sloka about this: 'In the Madhyama grama, Pancama and Rsabha are Samvadi; but in the Sadja grama, Pancama and Sadja are Samvadi'" (Rangacharya 2010, 219). As shown in Figures 11 and 12, these problems do not occur when the *sruti*-s are placed below their respective *svara*-s, in which case Ri and Pa are *samvadi* in the *madhyama grama*, while Ri/Ga and Dha/Ni are two-*sruti*, *vivadi* pairs in both *grama*-s.

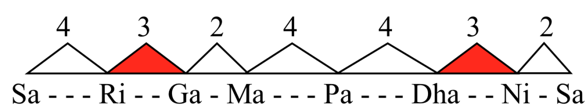

Figure 9. Kolinski, Hornbostel, and Fox Strangways's distribution of *sruti*-s above *svara*-s in the *shadja grama.*

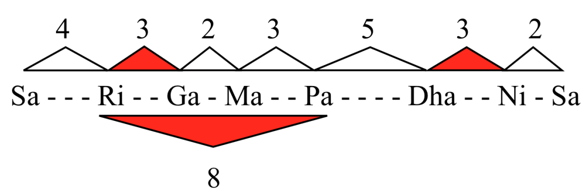

Figure 10. Kolinski, Hornbostel, and Fox Strangways's distribution of *sruti*-s above *svara*-s in the *madhyama grama.*

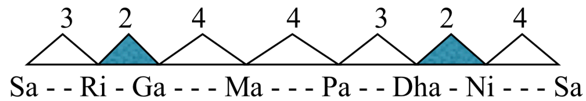

Figure 11. Distribution of *sruti*-s below *svara*-s in the *shadja grama* matches Bharata's description of *vivadi* (two-*sruti*) placement.

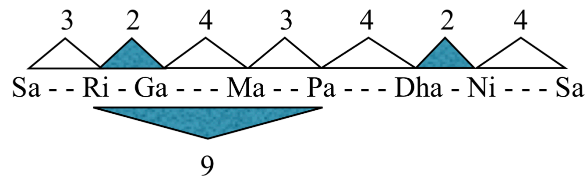

Figure 12. Distribution of *sruti*-s below *svara*-s in the *madhyama grama* matches Bharata's description of *vivadi* and *samvadi* (nine- or thirteen-*sruti*) interval placement.

10. A process called *svara sadharana* occurs when Ga or Ni is raised by two *sruti*-s, in which case they are called *antara gandhara* and *kakali nishada*, respectively. Scalar modulation, called *sadharanakrta*, and a modulated form of the *madhyama grama* result from this type of *svara* modification, and it may have been the origin of Bharata's system of modal scales or *murchana*-s (for *murchana*, see Rangacharya 2010, 220–21; for *svara sadharana*, *antara gandhara*, *kakali nishada*, and *sadharanakrta*, see Rangacharya 2010, 222–23).

11. Ma is considered to be the most important note that can never be left out. "Any svara may be left out . . . but the Madhyama can never be left out. . . . Madhyama is the most superior and indestructible" (Rangacharya 2010, 225). In the *Natyashastra*, this concept manifests in the formulation of pentatonic and hexatonic pitch sets, in which all other notes but Ma are omitted in various permutations (see Rangacharya 2010, 222; Bhandarkar 1912, 254–55). Contemporary Hindustani and Carnatic music differs significantly in that Sa is the only note that may never be completely removed from a scale.

12. Though Ma was considered to be indispensable in Bharata's music, it was not necessarily the tonic pitch by default—any *svara* could function as the starting point of a scale or mode in *Gandharva* music. This further contrasts with Hindustani and Carnatic music, in which Sa is always considered to be the modal tonic. Bharata's description of melodic forms, called *jati*-s, makes it clear that the choice of particular starting (*graha*), ending (*nyasa*), and emphasized (*amsa*) *svara*-s delineates modal identities in his system (see Rangacharya 2010, 225–30). In other words, the perceived tonic pitch probably modulated between the various *svara*-s of the harp-*vina* depending on the *jati* being performed. This helps to explain some elements of the *Natyashastra* that are hard to accept by current standards, such as the fact that the interval between between Sa and Pa in the *madhyama grama* is narrower than the *samvadi* interval by one *sruti*.

The *Samvadi* Interval

The *samvadi* or "consonant" interval (Ghosh 1961, 6–7) is of paramount importance, and subsequent conclusions regarding the tonal content of the *Natyashastra* depend upon its definition. Bharata described two *samvadi* intervals when he wrote: "those *svara*-s which are at an interval of nine or thirteen *sruti*-s from each other are mutually *samvadi.*" He provided a list of *samvadi* pairs (Sa/Ma, Sa/Pa, Ri/Dha, and Ga/Ni) all of which contain either nine or thirteen *sruti*-s, as shown by his *sruti* sequence for the *shadja grama* (Rangacharya 2010, 219). When combined, the two *samvadi* intervals complete the twenty-two-*sruti* octave and are thus intervallic inversions of each other. For example, an interval of nine *sruti*-s is found between Sa and Ma, and thirteen *sruti*-s are found between Ma and the octave of Sa, as illustrated in Figures 13 and 14. Likewise, Figures 15 and 16 show that thirteen *sruti*-s are found between Sa and Pa, and nine *sruti*-s are found between Pa and the octave of Sa.

A range of possible cent values for the *samvadi* intervals can be determined by stacking hypothetical nine- and thirteen-*sruti* intervals of various widths. For the sake of the following argument, a thirteen-*sruti samvadi* will be labeled as *samvadi* A*,* and a nine-*sruti samvadi* will be labeled as *samvadi* B. If a twelve-hundred-cent octave has twenty-two *sruti*-s and two consecutive *samvadi* A intervals add up to the twenty-sixth *sruti,* then two stacked *samvadi* A intervals must be wider than 1200 cents and one *samvadi* A interval must be wider than 600 cents, as shown in Figure 17. Similarly, the sum of three *samvadi* A intervals is thirty-nine

9

Sa - - Ri - Ga - - - Ma - - - Pa - - Dha - Ni - - - Sa

Figure 13. Nine-*sruti* interval between Sa and Ma.

13

Sa - - Ri - Ga - - - Ma - - - Pa - - Dha - Ni - - - Sa

Figure 14. Thirteen-*sruti* interval between Ma and the octave of Sa.

13

Sa - - Ri - Ga - - - Ma - - - Pa - - Dha - Ni - - - Sa

Figure 15. Thirteen-*sruti* interval between Sa and Pa.

9

Sa - - Ri - Ga - - - Ma - - - Pa - - Dha - Ni - - - Sa

Figure 16. Nine-*sruti* interval between Pa and the octave of Sa.

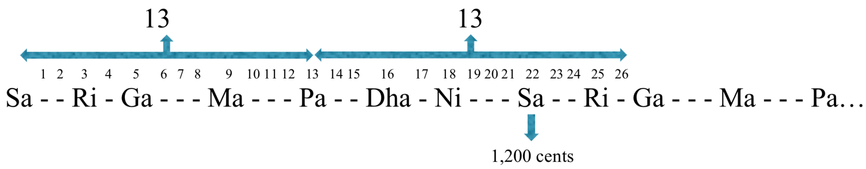

Figure 17. Two stacked *samvadi* A intervals.

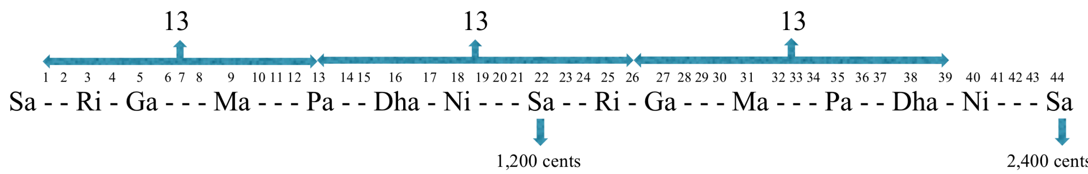

Figure 18. Three stacked *samvadi* A intervals.

sruti-s, which is narrower than the double octave found at the forty-fourth *sruti*. Therefore, three *samvadi* A intervals must be less than 2400 cents and one *samvadi* A interval must be less than 800 cents, as in Figure 18. When these calculations are extended up to the string limit of Bharata's harp-*vina* (a maximum of nine), the range for the *samvadi* A and B intervals can be narrowed to within 685 to 720 and 480 to 514 cents, respectively.

Because Bharata wrote that two *svara*-s in a *samvadi* relationship should "harmonise with each other" (Rangacharya 2010, 220) and because there is no evidence that musicians of his time tuned using a monochord, string length division, or advanced mathematical calculations (Forster 2010, 542–43; Widdess 1995, 7), it is probable that the *samvadi* intervals were tuned by ear using the most consonant pitches available. The most consonant intervals within the ranges previously determined for the *samvadi* A and B intervals are the perfect fifth (3:2, or 702 cents), and its inversion, the perfect fourth (4:3, or 498 cents).[6]

Such an approach to the *samvadi* interval is consistent with the tuning theory of other epochs in south Asian history and suggests that the interval between *vadi* and *samvadi* is the same today (in most cases) as it was nearly two thousand years ago. The ancient definition by *sruti* was reiterated in other texts, such as the *Brhaddesi* of Sri Matanga Muni (Sharma 1992) until approximately 1200 CE, when it last appeared prominently in the *Sangitaratnakara* of Sarngdeva (Shringy 2007, 115–59). A mathematically verifiable definition of the *samvadi* interval as interval ratios 3:2 and 4:3 stretches back to 1550 CE. Then, in a treatise titled *Svaramelakalanidhi,* Ramamatya consistently described fourths and fifths between the parallel

6. Simple-interval ratios, such as 3:2, 4:3, and 5:4, are frequently acknowledged by researchers as playing an important role in musical perception due to their consonant nature and are particularly important in this analysis of Bharata's music (Benade [1976] 1990, 274; Plomp and Levelt 1963, 3). The ratios 3:2 and 5:4 are derived directly from the third and fifth partials of the harmonic series by octave equivalence. Recognition of fundamental pitches, intervals, consonance, and scales often correlates highly with at least the first five members of the harmonic series (Oxenham 2013; Thompson 2013; Pierce 1983, 18–37, 62-64; Rasch and Plomp 1982, 5–6; Burns and Ward 1982, 257–258, 264; Benade [1976] 1990, 266–77; Terhardt 1974, 1066).

frets of adjacent strings as *samvadi* during his explanation of fret positions for the *rudra vina* (Forster 2010, 568–76; Aiyar 1932). In contemporary Hindustani music *shuddha* ("natural") Ma and Pa are tuned one fourth and fifth above Sa, respectively. Furthermore, the distance between *svara*-s labeled *vadi* and *samvadi* is usually a perfect fourth or fifth as well (Burns and Ward 1982, 257–58; see also Deva 1984). For example, of the ten *raag*-s described in detail in *The Classical Music of North India*, nine have *vadi–samvadi* pairs that are separated by perfect fifths or fourths (the exception is a major third in *raag Marwa*; Khan and Ruckert 2004).

The 3:2 and 4:3 *samvadi* intervals bring into harmony not just their respective constituent *svara*-s, but the rest of Bharata's statements as well. Many interpretations have agreed on this fundamental premise (Danielou 2010, 21–63; Forster 2010, 540–64; Sambamurthy 1999; Clements 1913, 49–53; Bhandarkar 1912). With the *samvadi* interval defined, the pitch content of Bharata's *sruti*-s can be analyzed by following his set of rules to its logical conclusion and by subsequently comparing the outcome with results produced by alternative approaches.

The *Samvadi* Chain

Imagine once again that a musician of the ancient world is absorbed in the task of tuning a harp-*vina* by ear. They have seven strings to work with and must tune as simply and efficiently as possible, probably by making only one adjustment per string, due to the inherent difficulty of tuning their instrument. It seems likely that musicians would have used the *samvadi* interval as a guide, in part because Bharata described it as a harmonizing interval, and in part because his initial description of the difference between *grama*-s was based on comparing *samvadi* pairs (Rangacharya 2010, 219–20). If so, a logical starting place would be the Pa-Sa-Ma-Ni-Ga chain.

Musicians may have begun by establishing Ma, because it is the central string of the seven-stringed harp-*vina,* as well as the central pitch of the *samvadi* group, and was considered by Bharata to be the "most superior and indestructible" *svara* (Rangacharya 2010, 225). Sa could then be tuned one *samvadi* perfect fourth below Ma. Another descending perfect fourth would situate Pa in the octave below Sa, but this would manifest as an ascending fifth by interval inversion to account for the proper sequence of strings and location of *svara*-s on the harp-*vina*. These initial tuning adjustments are shown in Figure 19. The inverse sequence,

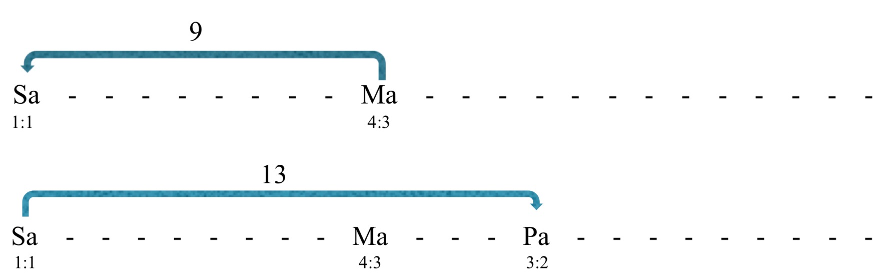

Figure 19. Tuning of Sa and Pa as successive *samvadi* perfect fourths descending from Ma.

consisting of two ascending perfect fourths above Ma, could also be tuned in order to establish Ni and Ga on the seventh and third strings, respectively. This would manifest as an ascending fourth, followed by a descending fifth (see Figure 20).

At this point, there are two ways in which the *samvadi* chain could be extended in order to determine values for Dha and Ri. First, Dha could be obtained from Ga by way of the nine-*sruti samvadi* interval (perfect fourth) and then Ri from Dha with a thirteen-*sruti samvadi* interval (perfect fifth), as shown in Figure 21. This procedure results in one form of *bhairavi thaat*, a Hindustani scale that correlates roughly with the Phrygian mode. However, the *sruti* sequence of this scale does not match the *shadja grama* as Bharata described it, in which Ri and Dha must rest three *sruti*-s above Sa and Pa, respectively.

The second possibility is to tune down nine *srutis* from Pa to Ri, and then up thirteen *sruti*-s from Ri to Dha. Figure 22 illustrates the outcome of this approach. This version of the

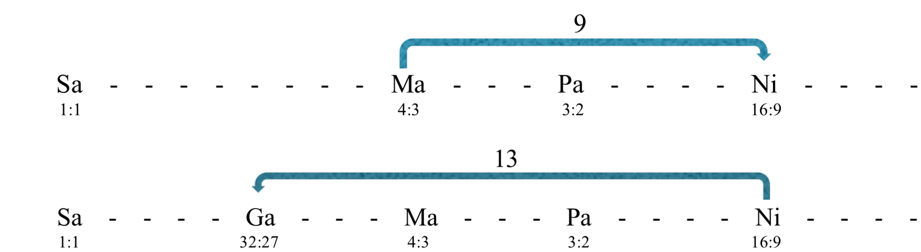

Figure 20. Tuning of Ni and Ga as successive *samvadi* perfect fourths ascending from Ma.

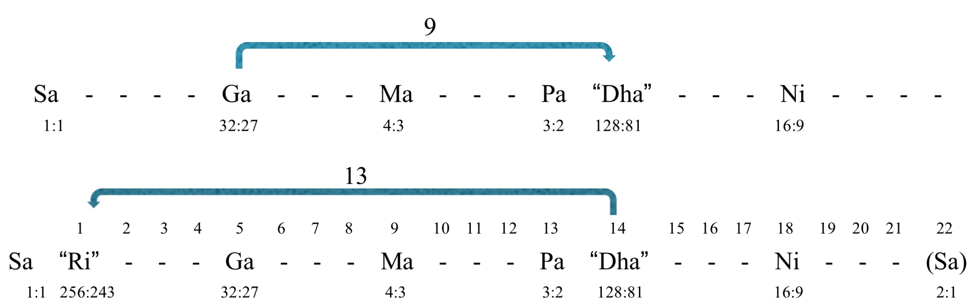

Figure 21. First method of tuning Dha and Ri, as successive *samvadi* intervals from Ga.

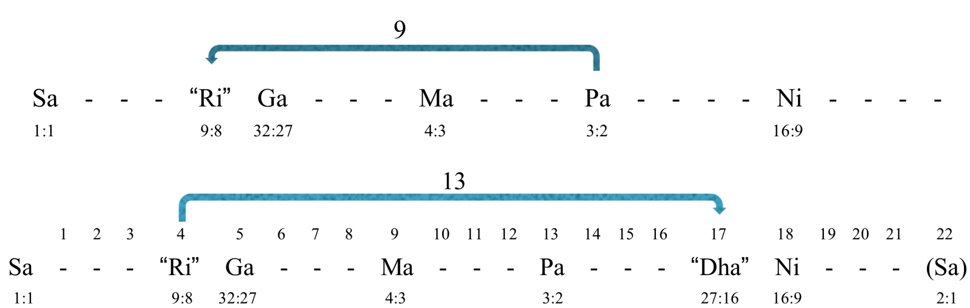

Figure 22. Second method of tuning Dha and Ri, as successive *samvadi* intervals from Pa.

Hindustani *kafi thaat*, similar to the Dorian mode, has a *sruti* count close to the one Bharata assigned to the *shadja grama*. However, its Ri and Dha *svara*-s are one *sruti* too high. It also produces a consonance between Ri and Pa, specifically forbidden in the *shadja grama* (Rangacharya 2010, 219).

In fact, no unbroken seven-note sequence of *samvadi* intervals can be aligned with the proper positions in the *shadja grama.* This is because, according to Bharata's distribution of *sruti*-s, Ri and Dha do not stand in a *samvadi* relationship with any note other than themselves and are not a contiguous part of the Pa-Sa-Ma-Ni-Ga chain. After carefully comparing the scales generated above using only *samvadi* intervals with Bharata's *shadja grama,* it seems likely that another interval was also used to tune the harp-*vina* to his specifications.

The Seven-*Sruti* Interval

> *"Dhivan is one who has got dhi (buddhi, intellect), the one related to him is dhaivata [Dha]… The place of dhaivata [Dha] is in the lalata (forehead), this is the meaning." – Brhaddesi* of Sri Matanga Muni, ca. 500 CE (Sharma 1992, 45)

Examining the numerous musical possibilities for Dha's elusive nature is a daunting but necessary task when interpreting the *Natyashastra*. Ultimately, it is possible to develop a comprehensive interpretation of the ancient *grama* system by incorporating the harmonic major third (interval ratio 5:4) as the seven-*sruti* span between Ma and Dha, as shown in Figure 23. Practically speaking, this can be attained by tuning one major third above Ma by ear. When combined with the *samvadi* chain, this approach produces the following seven-step tuning procedure for the harp-*vina*:

1. Tune the fourth string to a starting pitch, Ma.
2. Tune the first string one perfect fourth below Ma, to Sa.
3. Tune the seventh string one fourth above Ma, to Ni.
4. Tune the third string one perfect fifth below Ni, to Ga.
5. Tune the sixth string one major third above Ma, to Dha.
6. Tune the second string one fifth below Dha, to Ri.
7. Tune the fifth string one fifth above Sa to Pa, or one fourth above Ri to a low Pa instead, depending on the *grama*.[7]

However, several other theories must also be compared in order to confidently arrive at this conclusion.

7. This procedure can easily be modified to begin on any note without changing the number or nature of intervals in the requisite steps.

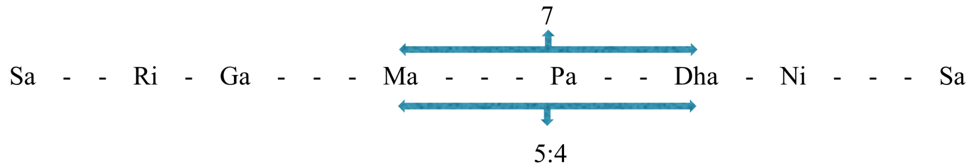

Figure 23. Seven-*sruti* major third between Ma and Dha.

The approximate range within which Dha must fall can be determined by re-examining two nearby positions: the nine-*sruti samvadi* above Ga and the "Pythagorean" (i.e., three-limit) major sixth that rests eight *sruti*-s above Ma. In the previous section, both pitch positions were derived using *samvadi* chains, and their relationship to Bharata's Dha is shown in Figure 24. Figure 24 makes it clear that the upper pitch of a Pythagorean major sixth (27:16) is not Dha of the *shadja* and *madhyama grama*-s because it rests eight *sruti-s* above Ma instead of the seven *sruti*-s indicated by Bharata's descriptions. This eight-*sruti* interval is equivalent to a Pythagorean major third (81:64), about 22 cents wider than a 5:4 major third. While it is not found between Ma and Dha, the eight-*sruti* major third is still a part of the *grama*-s and is found between Ga (32:27) and Pa (3:2) of the Pa-Sa-Ma-Ni-Ga *samvadi* chain (see Figure 25).

Still, there has been some disagreement regarding the application of the Pythagorean major sixth (27:16) to Bharata's *grama*-s. This is partly because some authors, like Kolinski (1961, 3–4), Hornbostel (quoted in Kolinski 1961, 4), and Fox Strangways (1935, 690–91) interpret Bharata's *sruti* sequence in such a way that the *sruti*-s follow their respective *svara*-s, instead of

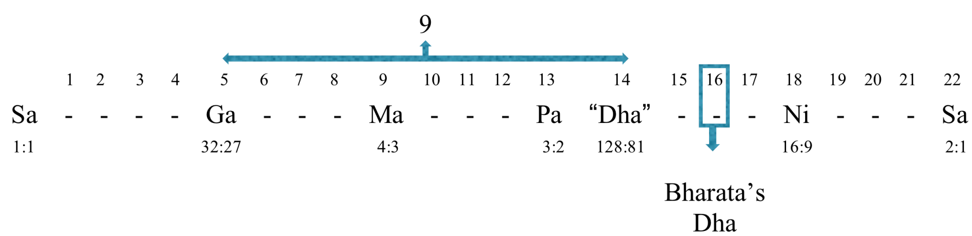

Figure 24a. The upper note of the interval ratio 128:81 rests one nine-*sruti samvadi* above Ga, on the fourteenth *sruti.*

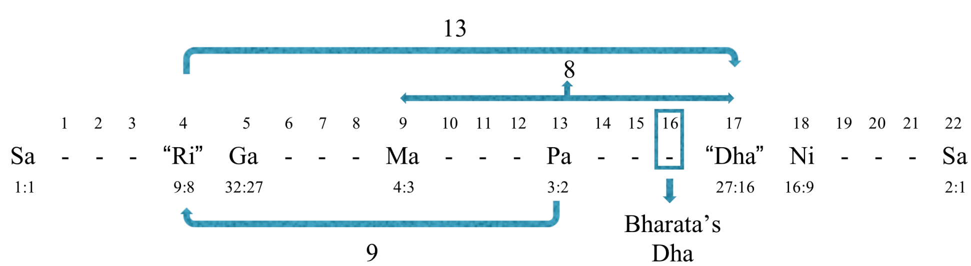

Figure 24b. The upper note of the interval ratio 9:8 rests on the fourth *sruti*, a 9-*sruti samvadi* interval below Pa. The upper note of the interval ratio 27:16, the Pythagorean major sixth, rests on the seventeenth *sruti*, a thirteen-*sruti samvadi* interval above 9:8, and eight *sruti*-s above Ma.

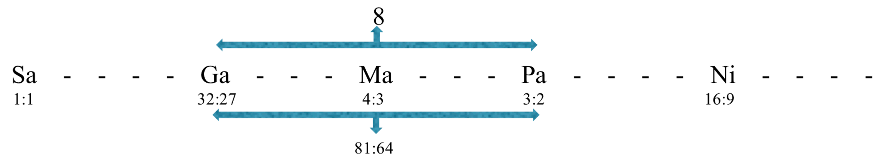

Figure 25. The eight-*sruti* major third with a ratio of 81:64 between Ga and Pa.

preceding them. This approach renders the *shadja grama* as a form of the major scale, which has significant impact on subsequent determinations regarding the nature of the constituent intervals, including the seven-*sruti* interval between Ma and Dha. However, as was previously concluded, this approach conflicts with two of Bharata's defining rules regarding the construction of the *shadja grama*: that Ri/Pa must not be *samvadi* in relation to each other and that Ri/Ga and Dha/Ni must be separated by two *sruti*-s (see Figures 9–12).

On the other hand, Jairazbhoy groups *sruti*-s below their assigned *svara*-s, but suggests that it would have been more logical to tune the *grama*-s using an unbroken chain of *samvadi* intervals. This would involve tuning Ri to Pa by *samvadi* and then tuning Dha to Ri, also by *samvadi* (Jairazbhoy 1975, 43–44). This particular tuning procedure was explored in the previous section on the *samvadi* chain, where it was shown to produce Ri and Dha *svara*-s resting one *sruti* higher than Bharata describes them, and to include the Pythagorean eight-*sruti* major third between Ma and Dha (see Figures 22 and 24b). In other words, Jairazbhoy suggests that the *shadja grama* could have been rendered with a different sequence of *sruti*-s than Bharata's in order to avoid producing the dissonant fifths that are found between Ri and Pa of the *shadja grama*, and Sa and Pa of the *madhyama grama*.

Many highly consonant scales, such as the just major scale, built on simple interval ratios like 3:2 and 5:4, contain one or more dissonant fifths, sometimes called "wolf" fifths. In just major scales, this can occur between the second degree and the sixth degree. To avoid this, one could raise the sixth degree so as to bring it into tune with the second degree, but this would cause a different wolf interval to be produced between the sixth degree and the third degree. Wolf intervals can be avoided in equal-tempered and Pythagorean versions of the major scale, but at the cost of reduced consonance for all of the major thirds. The single dissonant fifth found in each of the *grama*-s could be a result of choosing 5:4 major thirds over less consonant options.

In addition to their natural occurrence in just scales, dissonant fifths are also explained by the modulatory nature of Bharata's music. Unlike contemporary Hindustani and Carnatic music, *Gandharva* music tonicized different strings of the harp-*vina* depending on the musical context and was probably not accompanied by a drone instrument (Widdess 1995, 7). This could seem strange to Hindustani and Carnatic musicians, who tend to avoid including a dissonant fifth between Sa and Pa*,* a feature of the *madhyama grama*. In Bharata's modulating system, though, the dissonant quality of such an interval would sound less prominent, particularly in any of the modes or melodies that do not tonicize Sa.

It is instructive to compare the consonance of the various thirds in question and their *samvadi* counterparts. Figure 26 lists the first five partials (in Hz) of the upper members of the 5:4 major third, 5:3 major sixth, Pythagorean major third of 81:64, and the Pythagorean major sixth of 27:16. The fundamental pitch has been arbitrarily established at 400 Hz, and the interval ratio in each case refers to the upper member of the interval, from which the partials and their frequencies are then derived. Here, the fourth partial of 5:4 and the third partial of 5:3, shown in bold, would not produce any beating because they coincide exactly with the fifth partial of the fundamental (1:1). In contrast, the fourth partial of 81:64 and the third partial of 27:16, highlighted in red, are both slightly higher than the fifth partial of the fundamental and would beat against it at a rate of 25 Hz.

This acoustic analysis provides support for the position that ancient musicians might not have included 27:16 in favor of the more consonant 5:3 Dha. The following passage from Benade further illuminates the difference in listener perception between "harmonic" 5:4 thirds (which generate a 5:3 Dha when added to Ma), 12-TET "major thirds," and the "Pythagorean" thirds of 81:64 (which generate a 27:16 Dha when added to Ma):

> It is an easily verifiable fact that if one sets up the two oscillators to give a frequency ratio of 1.25992 (corresponding to an equal-tempered interval of exactly 400 cents), instead of our experimentally verified 1.25000 (=5/4) ratio (386 cents), everyone notices the resulting beats, and all the musicians in the group will say that an out-of-tune (sharp) major third is being sounded. When I tell my musician experimenters that the 400-cent interval is the equal-temperament version of the major third, they typically react with skepticism and dismay. They respond in even more intense fashion to the extremely rough-sounding combination whose frequency ratio is 1.26563 (=81/64) which spans an interval of 408 cents. This particular ratio, which is the product of 2000-year-old arithmetical ingenuity, is called a Pythagorean third. (Benade [1976] 1990, 275)

Jairazbhoy (1975, 38, 43–44) argues that it "is impossible to say with any certainty" which third was included in the *grama*-s. Furthermore, he writes that "Just Intonation . . . involves the 'divisive' principle," where "the tones are determined on the basis of . . . string length," a method for which there was no evidence in Bharata's writing. Jairazbhoy's statements seem to

	1:1	5:4	5:3	81:64	27:16
1st Partial	400	500	666	506	675
2nd Partial	800	1000	1333	1012	1350
3rd Partial	1200	1500	**2000**	1518	**2025**
4th Partial	1600	**2000**	2666	**2025**	2700
5th Partial	**2000**	2500	3333	2531	3375

Figure 26. The first five partials, in Hz, of just and Pythagorean imperfect consonances.

be rooted in a belief that the 5:4 interval could not have been tuned by ear in the *grama* system. However, the frequency analysis of 5:4, its derivation from the harmonic series by octave equivalence, its frequent inclusion in lists of consonant intervals by researchers, and experimental examples such as those provided by Benade, suggest that 5:4, the most consonant major third, can be recognized without relying on strength-length division (Plomp and Levelt 1963, 1966; Benade [1976] 1990, 274–75).

Differences in consonance aside, the pitch produced by 27:16 was already shown to rest on the seventeenth *sruti* by a sequence of *samvadi* intervals, and the pitch produced by 128:81 was demonstrated to rest on the fourteenth *sruti* by another sequence, both of which are shown again in Figures 27 and 28 for convenience. Bharata's Dha was found on the sixteenth *sruti*, somewhere between 27:16 and 128:81. In a broad sense, this suggests that the seven-*sruti* interval between Ma and Dha was either a major or neutral third. His description of an experiment in which pitch levels on two *vina*-s are compared can serve as a useful tool in assessing whether or not twenty-two-tone equal temperament (22-TET), other quarter-tone systems, or five-limit just intonation provide viable explanations for the nature of the seven-*sruti* interval and the Dha and Ri *svara*-s.

The *Vina* Experiment

The *vina* experiment described by Bharata (see Rangacharya 2010, 220) serves a number of significant purposes, the most notable of which is that it defines the relative proximity of pitches in the *shadja grama*. This is because Bharata describes the order in which the strings of two harps would reach unisons if one set were lowered in stages. He explains that one *vina* should be tuned a single *sruti* lower than the other after the first step of the experiment, as

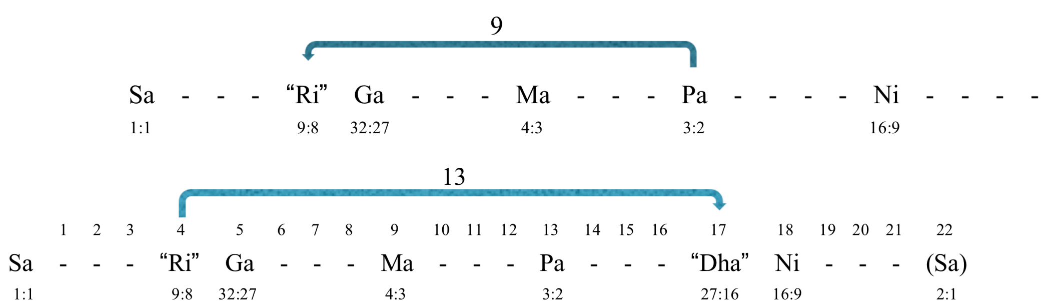

Figure 27. Derivation of the ratio 27:16 via successive *samvadi* intervals from Pa (cf. Figure 22).

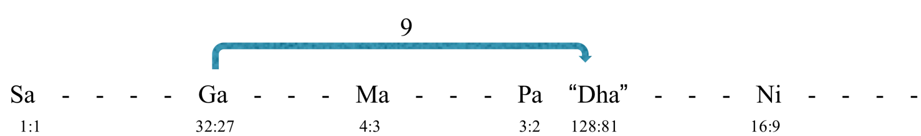

Figure 28. Derivation of the ratio 128:81 via a *samvadi* interval from Ga (cf. Figure 21).

shown in Figure 29. After two steps, Ni and Ga should arrive at Dha and Ri, as in Figure 30. Similarly, Ri and Dha should arrive at Sa and Pa after three steps, as in Figure 31. Finally, Sa, Ma, and Pa, should merge with Ni, Ga, and Ma after four steps, as in Figure 32 (Rangacharay 2010, 220). His explicit description of these points of alignment suggests that the only mode of the diatonic scale that could match the basic scale of the *shadja grama* is Dorian (*kafi*). In all the other modes, the pitches of the harps would align in a different order.[8]

To some authors, such as Jairazbhoy and Nijenhuis, the passage on the *vina* experiment demonstrates that Bharata perceived the twenty-two microtones in his system to be even (twenty-two-tone equal temperament or 22-TET). This is because, according to Jairazbhoy's translation, Bharata instructed readers to "lower again, in exactly this manner" the strings of the second *vina* on the second step (Jairazbhoy 1975, 41; see Rangacharya 2010 and Ghosh 1961

A		Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	-	Pa	-	-	Dha	-	Ni	-	-	-	-
B	Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	-	Pa	-	-	Dha	-	Ni	-	-	-	-	

Figure 29. *Svara*-s of two *vina*-s, labeled A and B, with *vina* B tuned one *sruti* lower than *vina* A.

A			Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	-	Pa	-	-	Dha	-	Ni	-	-	-	-
B	Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	-	Pa	-	-	Dha	-	Ni	-	-	-	(Sa)		

Figure 30. Equivalent pitches when *vina* B is tuned two *sruti*-s lower than *vina* A.

A				Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	-	Pa	-	-	Dha	-	Ni	-	-	-	-
B	Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	-	Pa	-	-	Dha	-	Ni	-	-	-	(Sa)			

Figure 31. Equivalent pitches when *vina* B is tuned three *sruti*-s lower than *vina* A.

A					Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	-	Pa	-	-	Dha	-	Ni	-	-	-	-
B	Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	-	Pa	-	-	Dha	-	Ni	-	-	-	(Sa)				

Figure 32. Equivalent pitches when *vina* B is tuned four *sruti*-s lower than *vina* A.

8. Additionally, the *vina* experiment provides some direct evidence that the *svara*-s spanned a complete octave, rather than another range such as a tetrachord. After being lowered four times, the Sa (first) string of one *vina* is supposed to align with the Ni (seventh) string of another, presumably seven pitches higher than Sa. This could only occur by octave equivalence on a seven-stringed *vina*.

for alternative translations). In other words, Jairazbhoy interpreted Bharata's instruction to mean that a single-*sruti* decrease of exactly the same interval must be generated at each step of the experiment. If attempted by ear alone, this can be accomplished by tuning Pa to Ri by 3:2 *samvadi*, bringing the *vina* into the *madhyama grama*, and then retuning the *vina* back to the *shadja grama* based on the new, lowered Pa, as shown in Figure 33. Consecutively repeating this one-*sruti* decrease would incrementally lower the pitches by the same amount each time, potentially proving that the *vina*-s would align in certain places after successive adjustments. However, the only system in which this procedure would produce the pitch-matching results described by Bharata is 22-TET, which does not incorporate 3:2 intervals.

The central dilemma of such an interpretation is that it remains unclear how a musician of his era would accurately tune a 22-TET scale on a seven-stringed harp-*vina* by ear in the first place. Pitches in 22-TET are, like in 12-TET, derived by applying logarithms. Figure 34 provides cent values for the 22-TET version of the *shadja grama*. Without access to the mathematical or technical ability to achieve such a system, a true 22-TET system is almost impossible to consistently tune by ear. Forster states this in the following passage:

> This equation [the formula for a 22-TET one-*sruti* interval] cannot be solved without logarithms. . . . The fact remains that a scientific or artistic experience of 22-tone equal temperament is impossible without advanced mathematics. No human being is able to accurately and consistently tune such a scale by simply listening and adjusting the tension of strings on a vina. . . .
>
> Since there is no historical evidence for the construction of monochords in ancient India, these "theories" could not have been realized in the tuning of ancient Indian

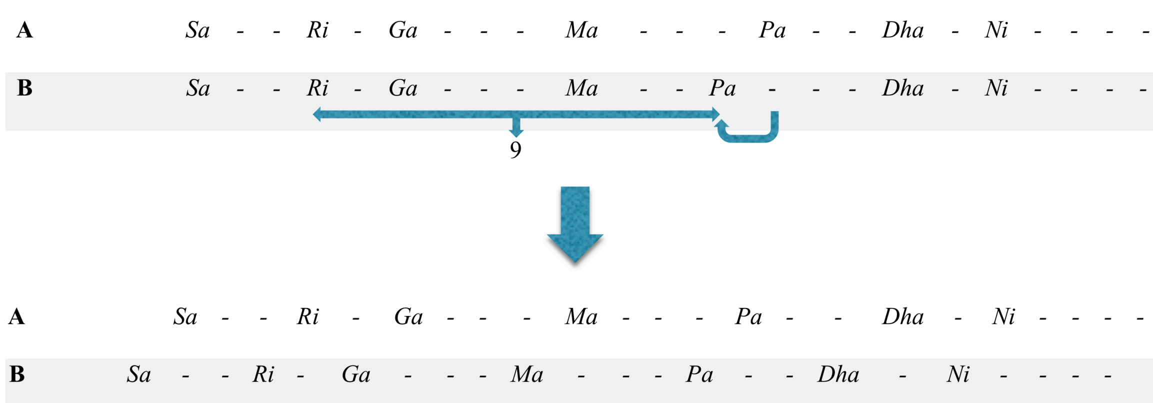

Figure 33. Procedure for lowering the pitch of *vina* B by lowering Pa to a *samvadi* interval above Ri.

Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	-	Pa	-	-	Dha	-	Ni	-	-	-	Sa
0 cents			163.6		272.7				490.86				709.02			872.64		981.72				1200

Figure 34. Cent values for the pitches of the *shadja grama* in 22-TET.

> instruments. . . . We conclude, therefore, that a literal interpretation of Bharata's pramana sruti [one-*sruti* interval between the Pa of *shadja grama* and that of *madhyama grama*] as an irrational interval ratio has no theoretical validity. (Forster 2010, 542–43)

Bhandarkar (1912, 256) argues that Bharata conceptualized a 22-TET tuning system as a way to approximately describe the underlying five-limit *kafi*, or Dorian scale of the *shadja grama*. In a true 22-TET system, the distance between Ma and Dha is extremely close to a 5:4 major third, differing only by four cents (see Bhandarkar 1912, 257–58). Similarly, the fourths and fifths in 22-TET are close approximations of those produced by using interval ratios 4:3 and 3:2.

Though he believed 22-TET was theoretically implied by the *vina* experiment, Jairazboy (1975, 41) concluded that "there is no possible way the *srutis* could have been equal, provided the concept of perfect fourths and fifths as we now understand it, was applicable in ancient India." Did Bharata know that the intervals in his system were uneven, or was he unaware of this property, as Jairazbhoy was suggesting? Perhaps it will never be known, but if 22-TET must be discarded as a practical option, Bharata could not have been applying an even one-*sruti* increment in his *vina* experiment while producing the results he described. If so, his true tonal system was uneven and he must have achieved the successive pitch alignments in some other way.

What is certain about Bharata's *vina* experiment is that he specifically described the points at which the *svara*-s are supposed to coincide as one *vina* is lowered in pitch. This sequence of pitch alignment can be used as a template to compare and assess the relevancy of various uneven interpretations of the *grama*-s. What follows is a suggested procedure for performing the *vina* experiment based on aligning the *svara*-s in the sequence that Bharata described, not by replicating an identical one-*sruti* decrease at each step. This demonstrates what the two *vina*-s would have sounded like at each step, according to his descriptions.

Step 1. Tune two harp *vina*-s, represented in Figure 35 as rows A and B respectively, to the *shadja grama*. *Vina* A functions as the control harp and remains at its original pitch level throughout the experiment, while *vina* B is lowered in pitch at each step.

Step 2. Subtract one *sruti* from Pa on the fifth string of *vina* B by tuning it to a *samvadi* relationship with the Ri on the second string. This effectively brings harp B into the *madhyama grama*. Then retune *vina* B's Sa string to its own Pa string, which has just been lowered. Finish by retuning the rest of *vina* B to the *shadja grama* following the standard intervallic relationships based on its new Sa and Pa. At this point, all of *vina* B has been lowered by the same interval that Pa descended, and therefore rests in a *shadja grama* tuning slightly lower than that of *vina* A. All of *vina* B is now out of tune with *vina* A. The procedure for step 2 is shown in Figure 36.

Step 3. Bharata specified that after step three, Ga and Ni of *vina* B would align with Ri and Dha of *vina* A. To accomplish this, retune the Ga and Ni strings on *vina* B to the Ri and

A *Sa* - - *Ri* - *Ga* - - - *Ma* - - - *Pa* - - *Dha* - *Ni* - - - -

B *Sa* - - *Ri* - *Ga* - - - *Ma* - - - *Pa* - - *Dha* - *Ni* - - - -

Figure 35. Initial, unison tuning of *vina*-s A and B in step 1.

A *Sa* - - *Ri* - *Ga* - - - *Ma* - - - *Pa* - - *Dha* - *Ni* - - - -

B *Sa* - - *Ri* - *Ga* - - - *Ma* - - *Pa* - - - *Dha* - *Ni* - - - -

9

A *Sa* - - *Ri* - *Ga* - - - *Ma* - - - *Pa* - - *Dha* - *Ni* - - - -

B *Sa* - - *Ri* - *Ga* - - - *Ma* - - - *Pa* - - *Dha* - *Ni* - - - -

Figure 36. Procedure for lowering *vina* B in step 2 (cf. Figure 33).

Dha strings of *vina* A and then retune the rest of *vina* B to the *shadja grama* based around the new Ga and Ni *svara*-s. Having also been lowered once before in step 2, these two strings would have been lowered twice each before coinciding with the strings of *vina* A. Step 3 is shown in Figure 37.

Step 4. According to Bharata, the next point of unison would occur when Ri and Dha of *vina* B reach the same pitch level as Sa and Pa of *vina* A. This can be accomplished by tuning the appropriate strings of *vina* B to unisons with those of *vina* A, followed by tuning *vina* B to the *shadja grama* based on its new Ri and Dha pitches. To arrive at this point, the second and sixth strings would have traveled three steps each before they matched with *vina* A. Figure 38 shows step 4.

A *Sa* - - *Ri* - *Ga* - - - *Ma* - - - *Pa* - - *Dha* - *Ni* - - - -

B *Sa* - - *Ri* *Ga* - - - - *Ma* - - - *Pa* - - *Dha* *Ni* - - - - -

A *Sa* - - *Ri* - *Ga* - - - *Ma* - - - *Pa* - - *Dha* - *Ni* - - - -

B *Sa* - - *Ri* - *Ga* - - - *Ma* - - - *Pa* - - *Dha* - *Ni* - - - -

Figure 37. Procedure for lowering *vina* B in step 3.

A *Sa* - - *Ri* - *Ga* - - - *Ma* - - - *Pa* - - *Dha* - *Ni* - - - -

B *Sa* - *Ri* - - *Ga* - - - *Ma* - - - *Pa* - *Dha* - - *Ni* - - - -

A *Sa* - - *Ri* - *Ga* - - - *Ma* - - - *Pa* - - *Dha* - *Ni* - - - -

B *Sa* - - *Ri* - *Ga* - - - *Ma* - - - *Pa* - - *Dha* - *Ni* - - - -

Figure 38. Procedure for lowering *vina* B in step 4.

Step 5. The final step is to bring Sa of *vina* B in line with Ni of *vina* A by octave equivalence, Ma of *vina* B with Ga of *vina* A, and Pa of *vina* B with Ma of *vina* A. Afterwards, all of *vina* B is retuned to the *shadja grama* based on the new Sa, Ma, and Pa. These strings would have travelled four steps each before coinciding with *vina* A. Step 5 is shown in Figure 39.

The *saung-gauk* may have descended from the ancient harp-*vina* (Becker 1967), and its traditional tuning can help to determine whether or not quarter-tone *grama*-s would produce the required results in Bharata's *vina* experiment. Robert Williamson (2010, 169–70) describes Burmese harp tunings that contain two disjunct chains of perfect fifths. The location of these fifths coincides with the placement of *samvadi* intervals in the *shadja* and *madhyama grama*-s when Sa is tuned to D in the Burmese system. According to Williamson (2010, 169–71), the traditional Burmese scales also contain intervals derived by the use of neutral seconds, approximately 150 cents wide; Williamson called these "Burmese seconds." Relative to D in the Burmese scale, these occur at the same positions as the *sruti*-s below Ri and the two versions of Pa. In figure 40, the *shadja grama* has been superimposed over the Burmese scale described by Williamson. Asterisks denote the placement of the Burmese seconds.

A *(Ni)* - - - *Sa* - - *Ri* - *Ga* - - - *Ma* - - - *Pa* - - *Dha* - *Ni* - - - -

B *Sa* - - - *Ri* - *Ga* - - *Ma* - - - *Pa* - - - *Dha* - *Ni* - - *(Sa)*

A *(Ni)* - - - *Sa* - - *Ri* - *Ga* - - - *Ma* - - - *Pa* - - *Dha* - *Ni* - - - -

B *Sa* - - *Ri* - *Ga* - - - *Ma* - - - *Pa* - - *Dha* - *Ni* - - - *(Sa)*

Figure 39. Procedure for lowering *vina* B in step 5.

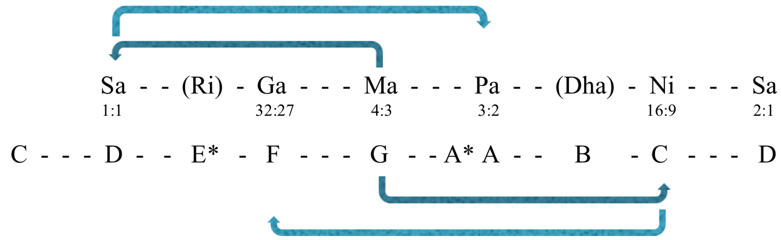

Figure 40. *Shadja grama* and Williamson's scale for the Burmese harp.

Williamson determined the width of Burmese seconds by analyzing frequencies tuned for their corresponding notes by several musicians. According to his research, two unnamed musicians produced Burmese seconds between C and B* that varied between 130 and 170 cents. On the other hand, musician U Myint Maung's intervals were found in the narrower range between 135 and 165 cents. In Maung's procedure, C was established first, and then B* was tuned beneath C by ear. Williamson (2010, 169–71) summarized his findings by stating that the Burmese second was found at 150 ± 15 cents, which matched Maung's tuning variance. The simplest interval ratio that is closest to the median of the range given by Williamson is 12:11, a neutral second of about 150 cents. Maung's tuning variance suggests that 12:11 bisects an interval range within which it is fairly difficult to tune notes precisely by ear. As a musician practiced in tuning pitches in this range, Maung's choices still deviated by 30 cents. Nevertheless, for the sake of argument, a 12:11 Burmese second can be interpolated between Ni and Dha in order to establish a quarter-tone *saung-gauk* tuning that can be tested for compatibility with Bharata's *vina* experiment.

The first three steps of the experiment will be mapped out below. Cent values are given for each of the *vina* strings. In step one (Figure 41), two *vina*-s are tuned to the hypothetical "Burmese *grama*" tuning based on Williamson's D pitch. In step 2 (Figure 42), *vina* B's Pa *sruti* is tuned to its Ri by *samvadi*, followed by an adjustment of the entire tuning to match this new Pa. In this case, Pa would descend by 60 cents total and the rest of the *svara*-s would follow suit. According to Bharata's instructions, *vina* B's Ni and Ga *svara*-s must align with *vina* A's Dha and Ri *svara*-s after step 3. Therefore, they can be adjusted to match, after which the rest of the *svara*-s can be brought into alignment with these new Ni and Ri pitch positions. This would cause a descent of 90 cents on each string of *vina* B. As can be seen in Figure 43, this adjustment causes a breakdown in the *vina* experiment because Ri and Dha of *vina* B dip below Sa and Pa of *vina* A. According to Bharata's instructions, these strings are supposed to

A	*0*	-	-	*144*	-	*294*	-	-	-	*498*	-	-	-	*702*	-	-	*846*	-	*996*	-	-	-	-
B	*0*	-	-	*144*	-	*294*	-	-	-	*498*	-	-	-	*702*	-	-	*846*	-	*996*	-	-	-	-

Figure 41. Starting pitches in cents for the *vina* experiment with "Burmese *grama*" tuning.

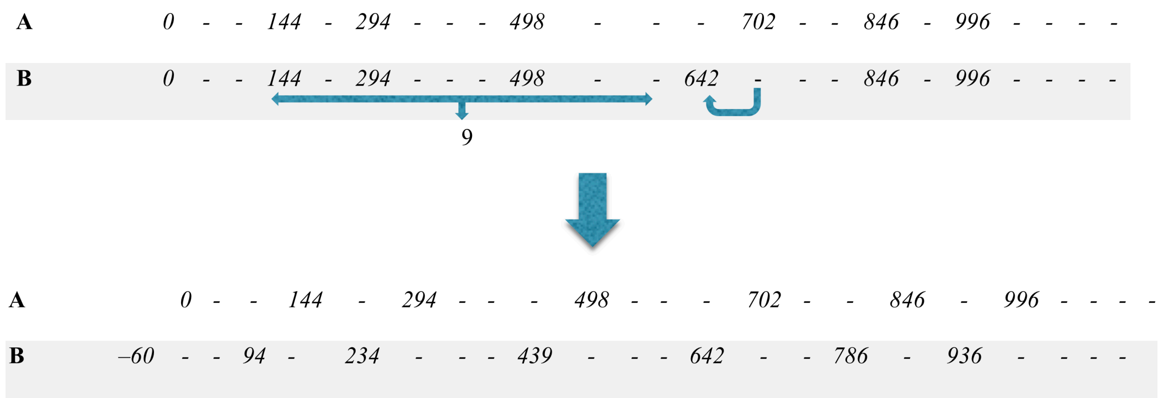

Figure 42. Step 2 of the *vina* experiment with "Burmese *grama*" tuning.

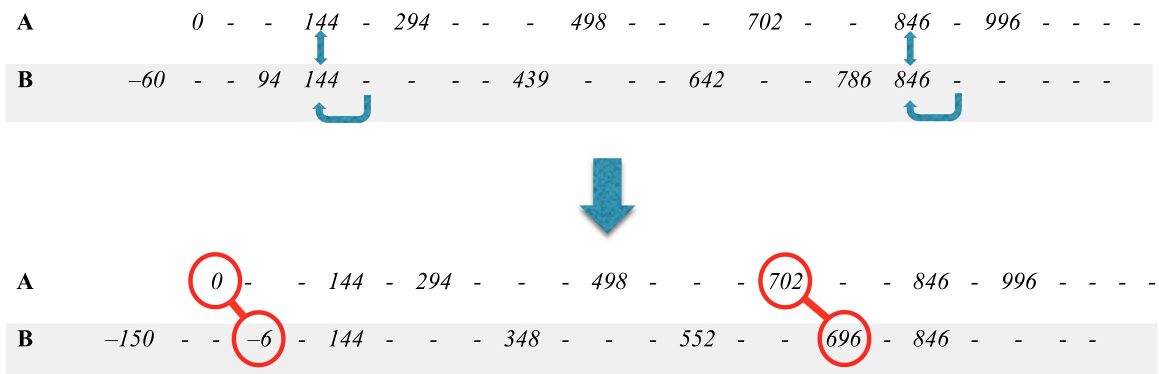

Figure 43. Step 3 of the *vina* experiment with "Burmese *grama*" tuning.

match after three decreases in *vina* B, and in this case, they have passed the mark after only two. This suggests that 12:11 quarter tones were not incorporated in the *grama*-s between Ni and Dha.

This approach to the *vina* experiment also establishes the lower limit for the value of Dha and Ri. If, as Bharata indicated, Ri and Dha must be higher than Sa and Pa after step three and Pa, Sa, Ma, Ni, and Ga are connected by a 3:2 *samvadi* chain, it can be concluded that Dha is higher than 853 cents and Ri must be higher than 151 cents. If tuned to this minimum value, Ri and Dha of *vina* B would rest one cent higher than the Sa and Pa strings of *vina* A, an extremely narrow musical interval that would not typically be perceived by listeners. Because such an interval would usually be heard as a unison, it is unlikely Bharata intended this (for perception of unisons, see Burns and Ward 1978; Sethares 2005, 43). Instead, the Ri and Dha *svara*-s of the *grama* system were probably found somewhere between these minimum values and the fourth and seventeenth *sruti*-s that were previously determined using *samvadi* chains (see Figure 24).

Emmie te Nijenhuis (1974, 16) provides a different neutral interval interpretation of Bharata's *grama*-s by including the interval ratio 11:10 (165 cents), as well as the very low 7:4 (969 cents) minor seventh. Her pitch choices are partly based on her position that Bharata's system closely resembled 22-TET. Her inclusion of 11:10 generates Ri and Dha pitches falling within the possible ranges indicated by the *vina* experiment. By incorporating 11:10 and 7:4, she is implying an eleven-limit system of just intonation.

Nijenhuis (1974, 16) also bases her choice of 7:4 (an octave equivalent of the seventh partial of the harmonic series) on the fact that Bharata provides a list of *samvadi* pairs which does not include Ma-Ni. She argues that Ma and Ni are not *samvadi* because of their exclusion, despite the fact that they are separated by nine *sruti*-s according to Bharata's other descriptions. Nijenhuis's choice to apply 7:4 between Ma and Ni creates two different definitions of the nine-*sruti samvadi* interval. Additionally, this causes the four-*sruti* interval between Ma and Pa to be distinguished from the four-*sruti* interval found between Ga and Ma, as well as Ni and Sa. When applied to the two methods Nijenhuis acknowledges for modulating between the *grama*-s, this intervallic inconsistency produces different microtonal versions of the *madhyama grama* itself, which are shown in Figure 44. In contrast to Nijenhuis's approach, multiple versions of the *madhyama grama* do not occur in five-limit or 22-TET interpretations.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
	Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	-	Pa	-	-	Dha	-	Ni	-	-	-	Sa
	1:1			11:10		7:6				4:3				3:2			33:20		7:4				2:1
cents:		165			102			231				204			165			102			231		

Figure 44a. Nijenhuis's *shadja grama*.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
	Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	Pa	-	-	-	Dha	-	Ni	-	-	-	Sa
	1:1			11:10		7:6				4:3			22:15				33:20		7:4				2:1
cents:		165			102			231			165				204			102			231		

Figure 44b. Nijenhuis's first method of deriving the *madhyama grama*, by lowering Pa into a nine-*sruti samvadi* relationship with Ri.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
	Ma	-	-	Pa	-	-	-	Dha	-	Ni	-	-	-	Sa	-	-	Ri	-	Ga	-	-	-	Ma
	1:1			11:10				5:4		4:3				3:2			33:20		7:4				2:1
cents:		165				221			112			204			165			102			231		

Figure 44c. Nijenhuis's second method of deriving the *madhyama grama*, by raising Ga by two *sruti*-s and changing the names of the *vina* strings accordingly. Intervals that differ between the two versions of the *madhyama grama* are circled.

Nijenhius (1974, 16) argues that the 7:4 interval could be tuned because it is "an easily recognizable interval when played in the third octave by a wind instrument." However, Bharata makes no mention of using a wind instrument as a tuning reference for the *vina.* In fact, he writes that "the svara-gandharva depends on the body (throat) and the *vina*," and that "the *svara*-s of the flute can be accomplished when the *vina* and the singer's body (throat) combine," suggesting that it may have been the other way around (Rangacharya 2010, 219, 245). Since Bharata was referring to the harp-*vina* when describing his tuning system, it is important to analyze the manner by which Nijenhuis's 11:10 interval could be tuned on a seven-stringed bow-harp.

Statements of experienced eleven-limit practitioners such as Harry Partch suggest that 11:10 would be significantly harder to tune on the *vina* than other simpler interval ratios. Partch devoted much of his life to composing and playing music in eleven-limit systems and offered an explanation for how this interval may be accessed by ear. His guide for tuning the Chromelodeon, a modified pump organ, indicates that 11:10 is to be achieved by initially tuning an 11:8 interval ratio above the fundamental frequency (Partch 1979, 139). This generates a pitch that is an octave equivalent of the eleventh partial, which is found in a quarter-tone position between the natural and sharp fourth degree. In Partch's approach, this interval must be tuned by ear, though it is rarely, if ever, included in the lists of consonant intervals provided by researchers such as Benade ([1976] 1990, 274) or Plomp and Levelt (1966). David Doty (2010) writes, "The harmonic tritone, 11:8, is harder to tune by ear, as the eleventh harmonic may be too weak for its beating to be easily detected. . . . It is probably best to use a programmable electronic tuner." Once 11:8 is established, Partch explains that a descent of a 5:4 major third produces the 11:10 neutral second above the fundamental. Though he offers a viable approach, assuming one can accurately tune 11:8 by ear, even Partch (1979, 126, 140) acknowledges that "the 7 and 9 identities are much stronger than 11. Since the importance of an identity in tonality decreases as its number increases . . . 11 is the weakest of the six identities of Monophony. . . . Ratios 11/8 and 16/11 are left until last. They are the least easy to tune." On a harp-*vina*, Partch's methods would require tuning one string to an 11:8 interval with Sa (assuming musicians could do so without the electronic tuner mentioned by Doty), then tuning the second string to a 5:4 major third below the pitch of the 11:8 string. To achieve Nijenhuis's *grama*-s, Dha could then be tuned to Ri by *samvadi*. Afterwards, the string tuned to 11:8 would have to be retuned to 4:3 with Sa, since 11:8 itself is not present in Nijenhuis's descriptions. Without further evidence to support that 11:10, when applied by ear, is a practical and consonant path to Ri or Dha, an 11-limit *grama* interpretation seems improbable on the ancient harp-*vina*.

Tuning difficulties aside, quarter-tone *grama* interpretations are still possible provided one accepts a greater degree of approximation. For aesthetic reasons, *Gandharva* musicians may have consciously chosen quarter-tone pitches rather than other, more consonant, choices. If they did, it may be that their Dha and Ri pitches were found within ranges close to those suggested by Nijenhuis. This approximate approach brings up another problem with neutral-interval interpretations of the *Natyashastra* in that the two-*sruti* and three-*sruti* intervals would sound approximately the same. If a two-*sruti vivadi* was practically indistinguishable from the three-*sruti* interval, it is unclear why Bharata emphasized the placement and importance of this interval and why he gave it a special name. In this case, the 294-cent span between Pa and Ni of the *samvadi* chain (see Figures 19–20) would be divided into a three-*sruti* interval of 135 to 165 cents, followed by a two-*sruti* interval of 129 to 159 cents, ranges with pitches either overlapping, or in very close proximity. It is possible that Nijenhuis decided to include interval ratio 7:4, a lower value for Ni, in order to narrow the width of the two-*sruti* interval and provide more distinction between the two, but as has been shown previously, this leads to other contradictions in various interval and *grama* definitions.

By way of contrast to 22-TET and the various neutral-interval interpretations, a system that incorporates one 5:4 major third between Ma and Dha, while elsewhere maintaining consistency of the 3:2 *samvadi* perfect fifth, provides an elegant solution to the *vina* experiment and to Bharata's *shadja* and *madhyama grama*-s. The most consonant pitch within the range established for Dha (853–905 cents) can be found by tuning one 5:4 major third above Ma by ear. Tuning such an interval would produce a Dha that rests 884 cents above Sa at interval ratio 5:3. Ri can then be tuned by *samvadi* to Dha, resulting in a pitch 182 cents above Sa at the interval ratio 10:9. Both of these steps are shown in Figure 45. The ten-*sruti* relationship between Ri and Pa shown in Figure 46 can then be checked by ear, revealing a noticeable dissonance, just as specified by Bharata for the *shadja grama*. To change to the *madhyama grama*, Pa is tuned by *samvadi* to Ri, rather than Sa, which reduces its position by one *sruti*, as in Figure 47. With another check by ear, the twelve-*sruti* interval between Pa and Sa shown in Figure 48 can be perceived to be slightly out of tune, which is exactly what Bharata prescribed for the *madhyama grama*.

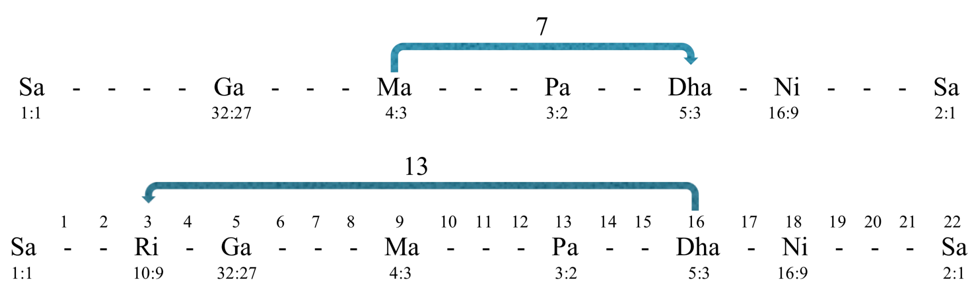

Figure 45. Tuning of Dha and Ri as a 5:4 major third above Ma and a *samvadi* interval below Dha in *shadja grama* (cf. Figures 21 and 22).

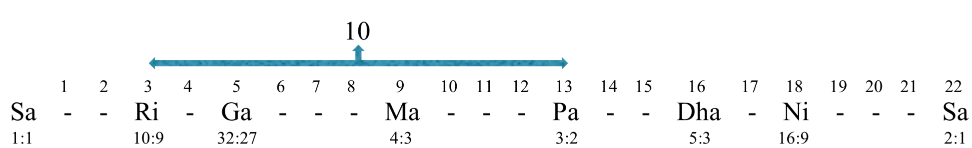

Figure 46. Dissonant ten-*sruti* interval between Ri and Pa in *shadja grama*.

9

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	Pa	-	-	-	Dha	-	Ni	-	-	-	Sa
1:1			10:9		32:27				4:3			40:27				5:3		16:9				2:1

Figure 47. Change from *shadja grama* to *madhyama grama* by tuning Pa a *samvadi* interval above Ri.

12

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	Pa	-	-	-	Dha	-	Ni	-	-	-	Sa
1:1			10:9		32:27				4:3			40:27				5:3		16:9				2:1

Figure 48. Slightly out-of-tune twelve-*sruti* interval between Pa and Sa in *madhyama grama*.

This *grama* interpretation fits Bharata's description of the *vina* experiment, producing the following series of pitch reductions (see Figure 49). In this case, the *svara*-s align in the sequence Bharata described without causing any of the overlapping pitch problems that can occur if Burmese seconds are included.

Given the relatively consonant nature of 5:4, as well as the caveats produced by alternative approaches, it is my conclusion that the *rishabha* and *dhaivata svara*-s of Bharata's music were probably tuned using one 5:4 major third. The interval ratios produced by a 5:4-inclusive approach to the *grama*-s of the *Natyashastra* are compiled in the Figures 50 and 51.

A	*0 - - 182 - 294 - - - 498 - - - 702 - - 884 - 996 - - - -*
B	*0 - - 182 - 294 - - - 498 - - - 702 - - 884 - 996 - - - -*
A	*0 - - 182 - 294 - - - 498 - - - 702 - - 884 - 996 - - - -*
B	*–22 - - 160 - 272 - - - 476 - - - 680 - - 862 - 974 - - - -*
A	*0 - - 182 - 294 - - - 498 - - - 702 - - 884 - 996 - - - -*
B	*–112 - - 70 - 182 - - - 386 - - - 590 - - 772 - 884 - - - -*
A	*0 - - 182 - 294 - - - 498 - - - 702 - - 884 - 996 - - - -*
B	*–182 - - 0 - 112 - - - 316 - - - 520 - - 702 - 814 - - - -*
A	*0 - - 182 - 294 - - - 498 - - - 702 - - 884 - 996 - - - -*
B	*–182 - - –22 - 90 - - - 294 - - - 498 - - 680 - 792 - - - -*

Figure 49. The steps of the *vina* experiment, with pitches based on tuning Dha a major third above Ma.

Svara	***Sargam***	***5-Limit Ratio***	***5-Limit Cents***	***12-TET Cents***	***22-TET Cents***
Sa	S	1:1	0	0	0
Ri	R	10:9	182	200	164
Ga	g	32:27	294	300	273
Ma	m	4:3	498	500	491
Pa	P	3:2	702	700	709
Dha	D	5:3	884	900	873
Ni	n	16:9	996	1000	982

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	-	Pa	-	-	Dha	-	Ni	-	-	-	Sa
1:1			10:9		32:27				4:3				3:2			5:3		16:9				2:1

steps in cents 182 112 204 204 182 112 204

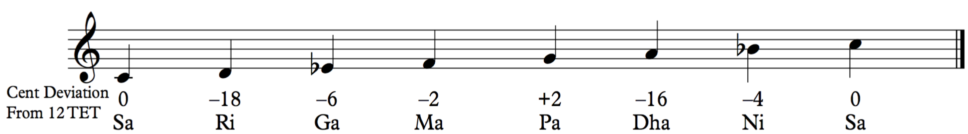

Figure 50. Profile of the five-limit *shadja grama* of the *Natyashastra*.

Svara	***Sargam***	***5-Limit Ratio***	***5-Limit Cents***	***12-TET Cents***	***22-TET Cents***
Sa	S	1:1	0	0	0
Ri	R	10:9	182	200	164
Ga	g	32:27	294	300	273
Ma	m	4:3	498	500	491
Pa	P	40:27	680	700	655
Dha	D	5:3	884	900	873
Ni	n	16:9	996	1000	982

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Sa	-	-	Ri	-	Ga	-	-	-	Ma	-	-	Pa	-	-	-	Dha	-	Ni	-	-	-	Sa
1:1			10:9		32:27				4:3			40:27				5:3		16:9				2:1

steps in cents 182 112 204 182 204 112 204

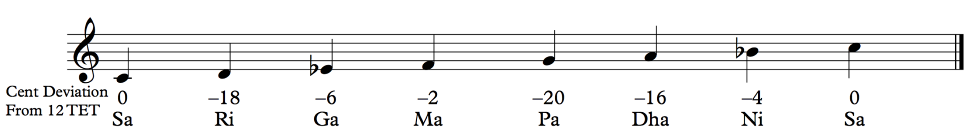

Figure 51. Profile of the five-limit *madhyama grama* of the *Natyashastra*.

Practically speaking, the five-limit *grama* systems can be tuned by ear on the harp-*vina* using a simple, seven-step tuning procedure, reprinted here for convenience, but this time beginning with the first string:

1. Tune the first string to a starting pitch, Sa.
2. Tune the fourth string one perfect fourth above Sa, to Ma.
3. Tune the seventh string one fourth above Ma, to Ni.
4. Tune the third string one perfect fifth below Ni, to Ga.
5. Tune the sixth string one major third above Ma, to Dha.
6. Tune the second string one fifth below Dha, to Ri.
7. Tune the fifth string one fifth above Sa to Pa, or one fourth above Ri to a low Pa instead.

Though any string could function as the tonal center of Bharata's scales, the tuning procedure described above would result in a series of notes that sounds very close to the modern Hindustani *kafi* scale (analogous to the Dorian mode) when considered from the lowest string of the instrument. The seventh mode of the *madhyama grama* (starting from Ni) corresponds to a five-limit major scale. The seventh mode of the *shadja grama* differs only in that its major sixth is twenty-two cents higher.

THE REMAINING *SRUTI*-S

In addition to the notes found in the *grama*-s, the microtonal positions of three other pitches are disclosed by both Bharata and Dattila. Relative to Sa, these outlying pitches include the major third (*shuddha* Ga), minor sixth (*komal* Dha), and major seventh (*shuddha* Ni), as listed in Figure 52. They are the product of another important feature of *Gandharva* music called *svara sadharana* or *sadharankrta,* a special modification of the tonal system that occurs when the *vina* strings corresponding to Ga and Ni are raised by two *sruti*-s. These two new pitches, called *sadharana svara*, bisect their neighboring *svara*-s in the *shadja* and *madhyama grama*-s and are named *kakali nishada* (*shuddha* Ni) and *antarasvara* or *antara gandhara* (*shuddha* Ga). The interpolation of *sadharana svara*-s shifts the *grama,* and all the

Contemporary Sargam	***Sadharana Svara***	***Ratio***	***Cents***	***12-TET***
G	*Antara Ga*	5:4	386	400
d	Unnamed	128:81	792	800
N	*Kakali Ni*	15:8	1088	1100

Figure 52. *Sadharana svara*-s.

associated *svara* names up or down by perfect fourths and fifths. This also causes the *grama* to switch back and forth between *shadja* and *madhyama*, a phenomenon described by both Bharata and Dattila.[9]

Dattila's first example of *svara sadharana* involves raising Ga of the *shadja grama* to the *antara gandhara* position, two *sruti*-s higher. This can be accomplished by retuning Ga to Dha by *samvadi* instead of its usual pairing with Ni and results in a pitch that rests seven *sruti*-s above Sa at interval ratio 5:4, as shown in Figure 53. This simple step changes the identity of Ga into Dha based on a new, modulated *madhyama grama*, in which Ma rests on the first string and Sa rests on the fifth string. In other words, the *svara* names of the harp strings have been shifted up by a perfect fifth.[10] Microtonally, this is the same *madhyama grama* scale that is produced by the method in which Pa is tuned to Ri by *samvadi*. The only differences are that the *svara*-s are located on different strings and in different registers. In the method that relies on lowering Pa, which is shown in Figure 54, Ma is still found on the central string of the harp and is thus at a higher pitch.

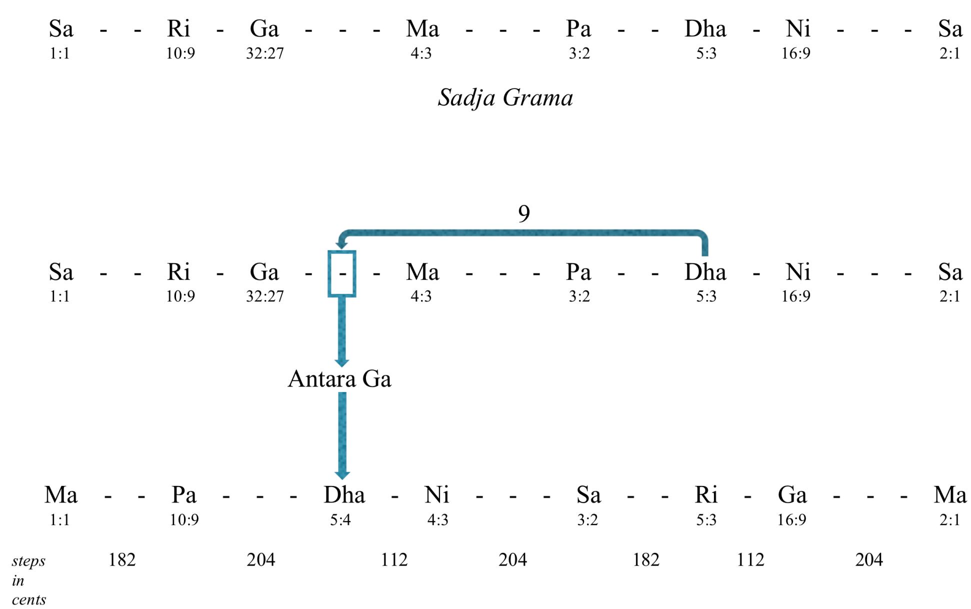

Figure 53. Tuning of the *madhyama grama* using *sadharana*.

9. "[B]y increasing two *sruti*-s, and changing Gandhara to Dhaivata, the murchana and the grama become different" (Rangacharya 2010, 221). "[I]n the madhyamagrama . . . Having changed dhaivata into gandhara by taking away two srutis, one may make madhyama and the other notes . . . murchanas of the sadjagrama" (Nijenhuis 1970, 21).

10. "One may change gandhara into dhaivata by raising it by two srutis, and in the same way, in due order, madhyama and the other notes into nisada, etc." (Nijenhuis 1970, 21). Dattila pointed out that in the new tuning, Ma is now equivalent to Ni, and that the rest of the notes follow suite, with Pa now being equivalent to Sa, and so on. Bharata gives the same example. "One Murchana can be produced in two ways; for example in the Sadja grama, by increasing two *sruti*-s, and changing Gandhara to Dhaivata" (Rangacharya 2010, 221). Both authors meant that Ga was to be tuned to the current Dha by *samvadi*, and at the same time that it literally "changes" into the Dha of the new, modulated *grama,* as a result.

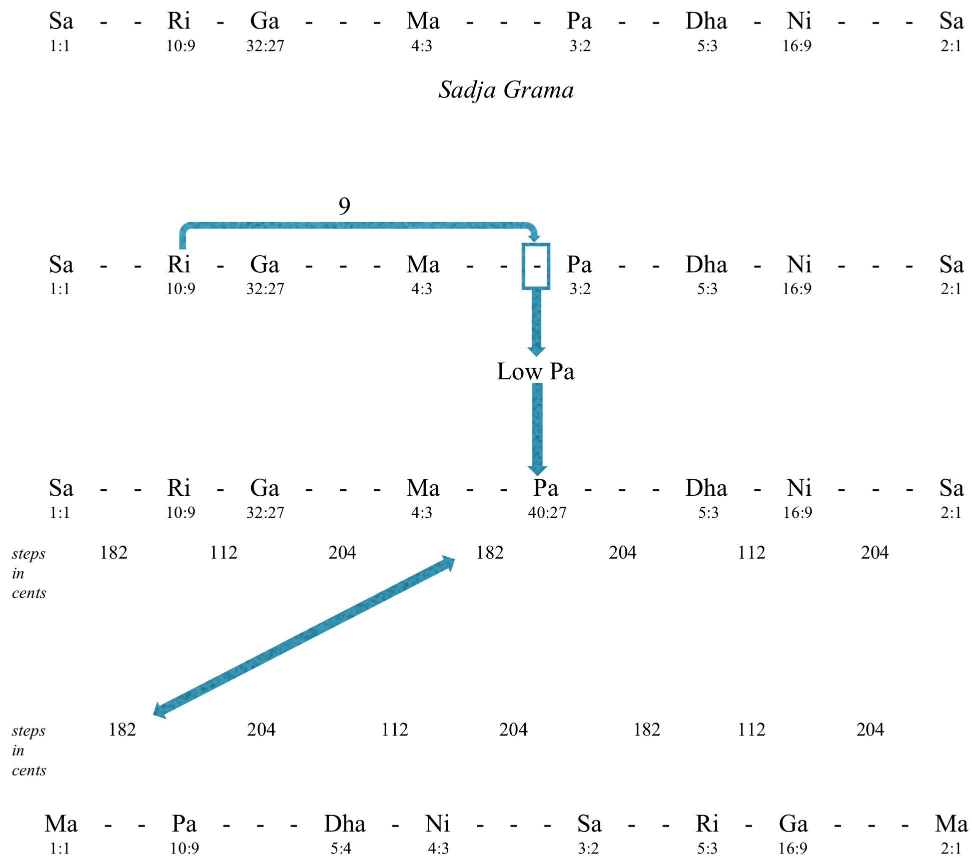

Figure 54. Tuning of the *madhyama grama* by lowering Pa, compared with the *madhyama grama* produced using *sadharana*.

Dattila also describes the inverse procedure, in which Dha of the *madhyama grama* is lowered by two *sruti*-s. This can be accomplished by tuning Dha to Ga by *samvadi*, and it causes the *svara*-s to relocate to different strings of the *vina*, shifting upwards by a perfect fourth.[11] Following the first *svara sadharana* modulation into the *madhyama grama*, this would effectively return the *vina* to its original *shadja grama* tuning, as shown in Figure 55.

It is possible that this simple operation, explained by both authors, represents the basic procedure for changing *grama*-s on the *vina*. The *grama* names themselves could be derived from the placement of the respective *svara*-s on the first string (i.e., because Ma, having been exchanged with Sa, is made to rest on the first string of the harp, it is therefore called the *madhyama grama*). Furthermore, if tuning via *svara sadharana* is central to the practice of Bharata's music, it provides a simple musical explanation for the origin of the entire *sruti* and *grama* concept. In other words, the subtle distinction between Pa of the *shadja* and *madhyama*

11. "Having changed dhaivata into gandhara by taking away two srutis, one may make madhyama and the other notes, in the same way as has been explained before, murchanas of the sadjagrama" (Nijenhuis 1970, 21). Bharata referred to the same procedure: "In the Madhyama grama too, the murchana can be produced in two ways: by Dhaivata being softened" (Rangacharya 2010, 221).

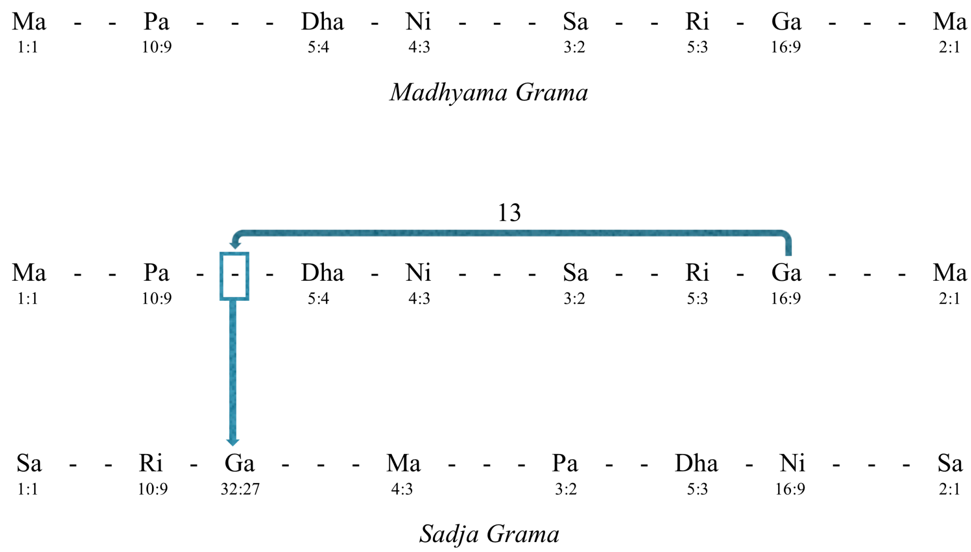

Figure 55. Returning from the *madhyama grama* to the *shadja grama* by lowering Dha.

grama-s occurs as the acoustic result of what may have been the standard method for switching between the two tonal systems on the harp-*vina*: tuning the third string to *antara gandhara* instead of the standard Ga (see also Widdess 1995, 206–9).

Beyond the *shadja* and *madhyama grama*-s, both Bharata and Dattila agree that there are as many *murchana*-s as there are notes in each *grama* (Rangacharya 2010, 221; Nijenhuis 1970, 19). The various *murchana*-s have traditionally been defined as the modes of each tuning system. This is because Bharata assigns each *murchana*, with its own particular name, to a specific *svara* of each *grama* (Rangacharya 2010, 221). However, he also states that changing notes through *sadharanakrta* modulates the *murchana* as well as the *grama* (Rangacharya 2010, 221), so it could be that a *murchana* was a modulated mode rather than a simple tonicization of a different mode in the standard tuning.[12] Given the connection between *murchana* and *svara sadharana,* it may be that the *murchana*-s were the result of cycling through the *sadharana* modulations previously described. With regards to the application of this possibility, it is unlikely that ancient musicians would take this process more than a few steps in either direction due to the practical limitations of the *vina*. If entirely modulated *murchana* tunings were incorporated into actual music, musicians probably established different strings of the harp as Sa or Ma and then proceeded with the standard tuning sequence, rather than performing a strict series of modulations starting in the *shadja grama*. While *murchana* theory is an interesting topic, its description in the *Natyashastra* does not offer further aid in determining *sruti*-s themselves.

12. Tonicized modes might be better described by the concept of *jati*, which delineates predetermined melodic tendencies that emphasize specific *svara*-s of each *grama.* Because of the inclusion of ascending and descending patterns and the precise treatment of the melodies, the *jati*-s may be ancient precursors to the contemporary *raag*-s. It is possible that *murchana*-s were used in conjunction with *jati*-s to establish the appropriate range in accordance with the particular requirements of a performance. For more details on *jati*, see Rangacharya (2010, 225–30).

Beyond the Texts

Dattila explains that only certain *sruti*-s, probably those designated *svara*-s or *sadharana svara*-s, were actually used in songs (Nijenhuis 1970, 19). Eleven in total, these have been derived through a direct examination of the texts, and are listed in Figure 56 by their contemporary *sargam* equivalents.

The remaining eleven *sruti*-s can be determined by carefully applying the *samvadi* and seven-*sruti* tuning intervals outside of the two *grama*-s. Because simple-interval ratios (such as 3:2 and 5:4) may be compounded infinitely without ever returning to the same frequency, any chain of such intervals extended far enough will inevitably clash with the finite limit imposed by Bharata's twenty-two *sruti* count. This is not a unique feature of his tuning system, but a shared feature common to all bounded systems of simple-interval ratios. The only way to truly achieve a finite, equally tempered system is to leave behind the more consonant interval ratios in favor of irrational fractions. However, for the reasons previously discussed, 22-TET is probably not an adequate explanation for Bharata's *sruti*-s.

The best way to determine the remaining eleven pitch positions is to fill in the gaps as logically as possible, by tuning chains of consonant intervals without exceeding the seven-string limit of the ancient *vina*.[13] Because nine-*sruti samvadi*-s are interval inversions of their thirteen-*sruti* counterparts, there are only two *samvadi* sequences, directionally speaking. The first is a series of ascending fifths (equivalent to descending fourths) and the second a series of descending fifths (equivalent to ascending fourths). However, in order to generate a seven-note scale within the span of one octave, as is appropriate on the ancient *vina*, the pitches found in the continuously ascending and descending sequences described above must be modified by octave equivalence. As a result, each sequence, adjusted accordingly, manifests on the seven strings of the harp as a mixture of fourths and fifths applied in various directions.

The sequence of ascending fifths, starting on the middle (fourth) string of the seven-string *vina* and adjusted by octave equivalence, generates the seven *svara*-s shown sequentially in Figure 57. Similarly, Figure 58 demonstrates a series of descending fifths starting from Sa, which adds five more pitches. The introduction of the seven-*sruti* interval of 5:4 above Ma produces Dha. Ri, the low Pa of the *madhyama grama*, and the two *sadharana svara-s*, *antara gandhara* and *kakali nishada*, are all connected to Dha by another *samvadi* chain, shown in Figure 59. Collectively, then, Figures 57 to 59 generate seventeen different *svara*-s.

S	-	-	R	-	g	-	G	-	m	-	-	P	P	d	-	D	-	n	-	N	-	S
1:1			10:9		32:27		5:4		4:3			40:27	3:2	128:81		5:3		16:9		15:8		2:1

Figure 56. The eleven *sruti*-s derived from the texts by Bharata and Dattila.

13. Even within the seven-string limit, some *sruti* positions exhibit more than one possibility depending on the specific interval combination employed. The *sruti*-s presented herein represent my best attempt to isolate the pitches that are most relevant to Bharata's descriptions of the *sruti* and *grama* system.

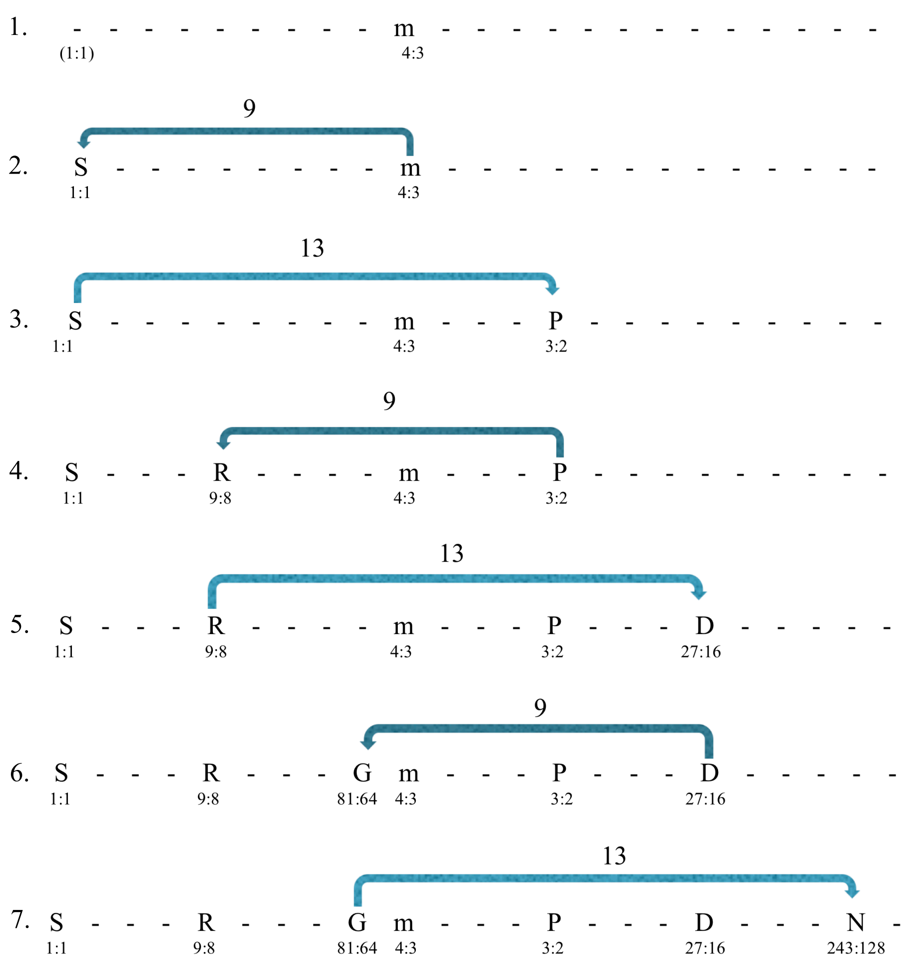

Figure 57. *Svara*-s generated from a sequence of ascending fifths.

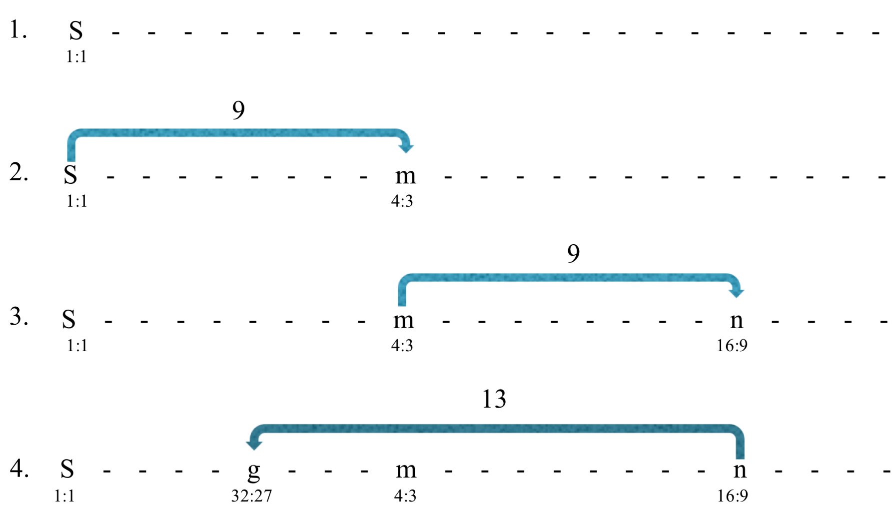

Figure 58. *Svara*-s generated from a sequence of ascending fourths.

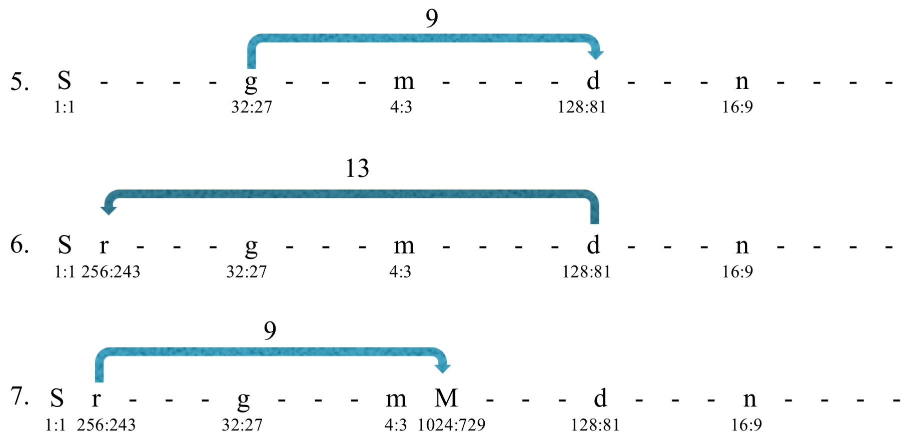

Figure 58 continued.

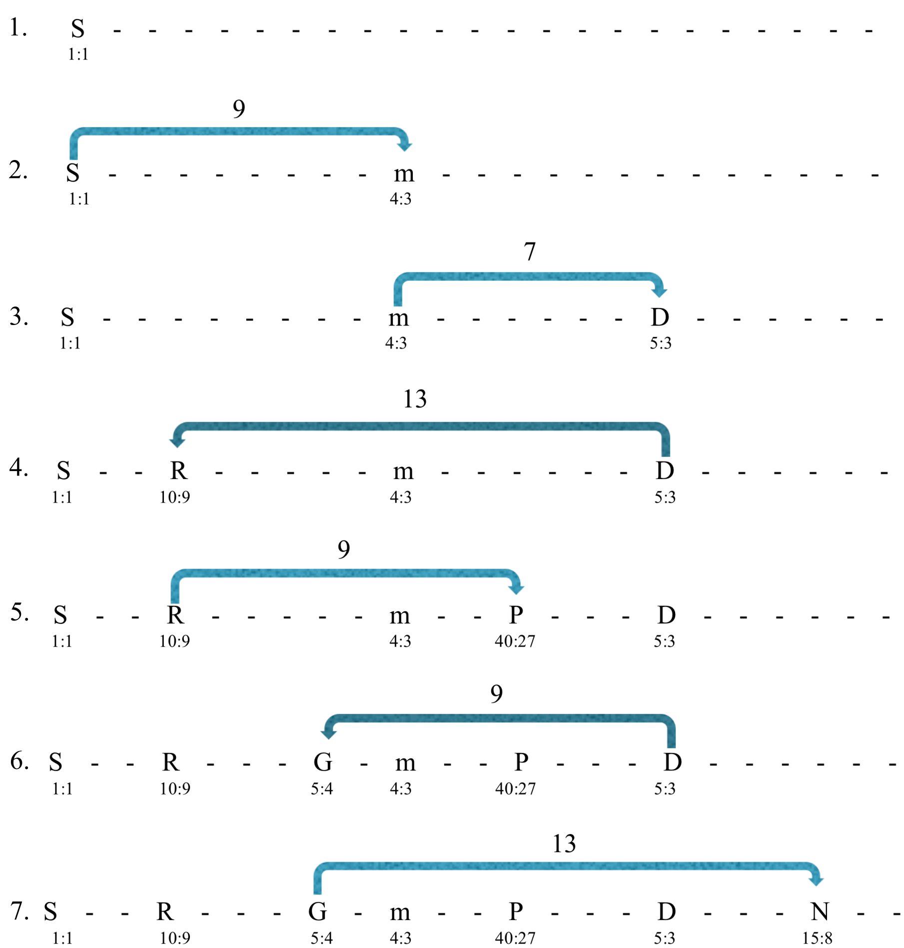

Figure 59. *Svara*-s generated from a *samvadi* chain that includes Dha a major third above Ma.

Just like perfect fifths, major thirds can also be applied in ascending and descending directions. The *samvadi* series in Figure 59 is linked to Dha, seven *sruti*-s above Ma, and the final *samvadi* series needed to reach the total of twenty-two *sruti*-s begins seven *sruti*-s below Ma. Tuning this note accordingly (one major third lower than Ma) produces a high form of the *komal* (flat) Ri. This pitch has an interval ratio of 16:15 in relation to Sa, which is equivalent to Bharata's *vivadi* interval as it appears between other note pairs in his system (such as Ri/Ga and Dha/Ni). The five missing *sruti* positions are directly tied to this pitch by a final chain of *samvadi* intervals, shown in Figure 60.

These four *samvadi* chains complete the list of twenty-two *sruti*-s given below in Figure 61. A way to visualize the intervallic relationships between each of the twenty-two *sruti*-s is shown in Figure 62. This constellation of *sruti*-s allows for quick visual comprehension of all interrelationships between the notes and also serves as a guide by which any tuning within the twenty-two-*sruti* system may be mapped out and understood. Moving upwards and to the right adds a major third, moving down and to the left subtracts a

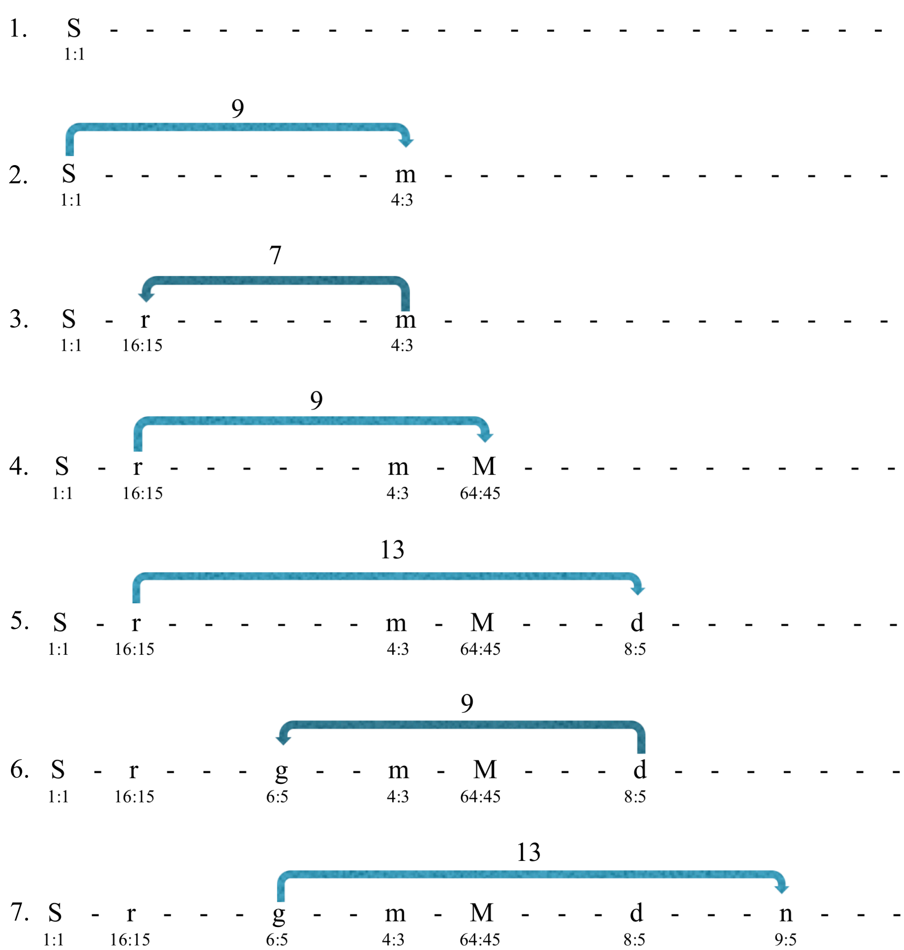

Figure 60. *Svara*-s generated from a *samvadi* chain that includes Ri a major third below Ma.

Sruti	Svara	Sargam	Ratio	Cents	12-TET
0	Sa	S	1:1	0	0
1		r	256:243	90	100
2		r	16:15	112	100
3	Ri	R	10:9	182	200
4		R	9:8	204	200
5	Ga	g	32:27	294	300
6		g	6:5	316	300
7	Antara Ga	G	5:4	386	400
8		G	81:64	407	400
9	Ma	m	4:3	498	500
10		M	1,024:729	588	600
11		M	64:45	610	600
12	Pa	P	40:27	680	700
13	Pa	P	3:2	702	700
14		d	128:81	792	800
15		d	8:5	813	800
16	Dha	D	5:3	884	900
17		D	27:16	905	900
18	Ni	n	16:9	996	1,000
19		n	9:5	1,018	1,000
20	Kakali Ni	N	15:8	1,088	1,100
21		N	243:128	1,109	1,100
22	Sa	S	2:1	1,200	1,200

Figure 61. The twenty-two *sruti*-s generated in Figures 57 to 60.

major third, moving up and to the left adds a perfect fifth, and moving down and to the right subtracts a perfect fifth. Green circles represent *svara*-s found within Bharata's two *grama*-s, while blue circles represent the three *sadharana svara*-s.

The pitches in the final list do not represent a scale comprising twenty-two even intervals. Furthermore, three types of one-*sruti* intervals arise incidentally from the application of 3:2 and 5:4 tuning intervals. It is best to think of the pitches presented here in pairs, as microtonal variations of the same pitch regions used in contemporary Hindustani and Carnatic music: there are two versions of "r" (the flattened second) and two versions of "R" (the natural second). Fox Strangways (1935, 692) traced this concept of tonal "twins" to an even more ancient document, the *Rkpratisakhya,* written around 400 BCE. For those who remain troubled by the two versions of Pa when attempting to apply Bharata's system to contemporary practice, the dissonant fifth (40:27) can be avoided by renaming Ma of the

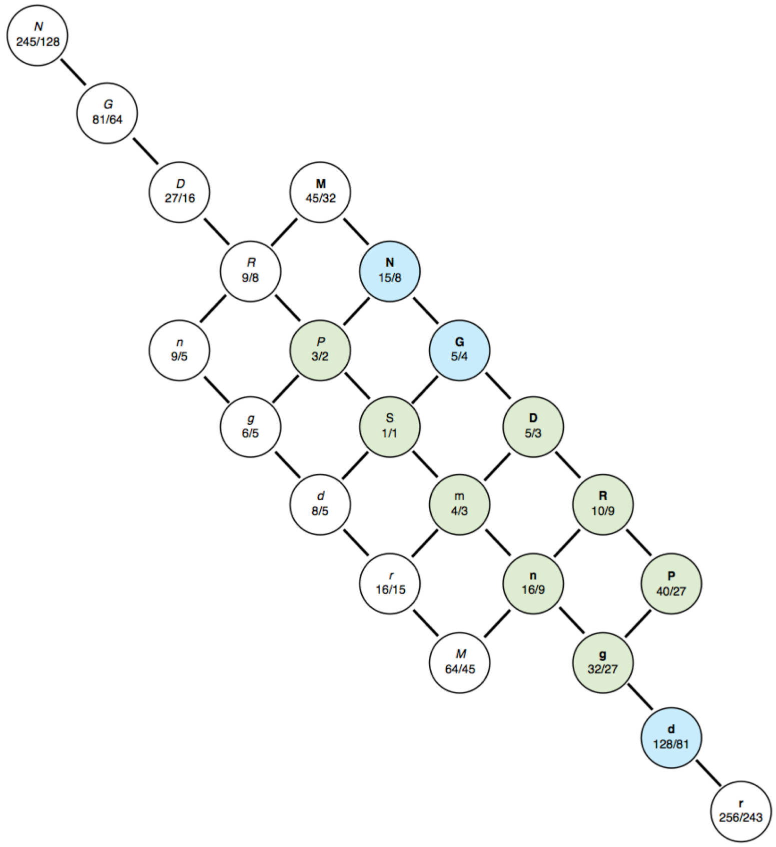

Figure 62. Constellation of *sruti*-s showing relationships of perfect fifths and major thirds.

ancient system "Sa" and transposing the other pitches similarly in sequence. This collection is exactly the same as Bharata's, but contains only one version of Pa and two versions of *shuddha* Ma (the natural fourth).

Tuning Five-Limit *Grama*-s on the Sitar

Before concluding, I would like to offer a simple method by which musicians can apply the *shadja* and *madhyama grama*-s to the sitar. This can be accomplished by tuning the harp-like sympathetic strings in note pairs. To do so, one must place their thumb beneath the main playing strings on the side with the high *chikari* strings, and then reach over top, placing the rest of their fingers beneath the main strings from the opposite side. This enables two sympathetic strings to be plucked simultaneously by the thumb and fore or middle fingers. The strings may then be tuned in a manner that mimics the way the ancient harp-*vina* may have been tuned, with slight modifications to accommodate for the particular string setup and array of notes on the sitar. The moveable frets of the *sitar* should be adjusted afterwards, so as to match the intonation of the strings. Figure 63 provides a tuning procedure for the *shadja grama* on a sitar with thirteen sympathetic strings.

1. Sa - - - - - - - - - - - -

1:1

2. Sa - Sa - - - - - - - - - -

3:2

3. Sa - Sa - - - Pa - - - - - -

1:1

4. Sa - Sa - - - Pa Pa - - - - -

4:3

5. Sa - Sa - - Ma Pa Pa - - - - -

4:3

6. Sa - Sa - - Ma Pa Pa - Ni - - -

3:2

7. Sa - Sa - Ga Ma Pa Pa - Ni - - -

5:4

8. Sa - Sa - Ga Ma Pa Pa Dha Ni - - -

3:2

9. Sa - Sa Ri Ga Ma Pa Pa Dha Ni - - -

2:1

10. Sa - Sa Ri Ga Ma Pa Pa Dha Ni Sa - -

2:1

11. Sa - Sa Ri Ga Ma Pa Pa Dha Ni Sa Ri -

2:1

12. Sa - Sa Ri Ga Ma Pa Pa Dha Ni Sa Ri Ga

2:1

13. Sa Ni Sa Ri Ga Ma Pa Pa Dha Ni Sa Ri Ga

Figure 63. Tuning the *shadja grama* on thirteen sympathetic strings of a sitar.

To those familiar with modern Hindustani music, one of these pitches might sound particularly odd. If tuned properly, the second degree should be slightly out of tune with Pa. This will clash somewhat with the ever-present Pa found in both the drone of the *tanpura* as well as the high-pitched *chikari* drone strings of the sitar. It is important to remember that there is no evidence for the use of drones in Bharata's time and that his music did not exclusively tonicize Sa. An alternative would be to tune so that Pa is de-emphasized and Ma becomes prominent ("Ma-*vadi*" tuning). In this case, the low Ri might be heard as a bit more palatable, sounding to Ma as Dha does to Sa, whereas Pa would sound like a low Ri in relation to Ma.

Recognizing that Sa was probably not the only tonic pitch in Bharata's music, another approach would be to apply the different modes of the *shadja* and *madhyama grama*-s to the sitar, maintaining the proper sequence of intervals in their transposed positions. For example, I sometimes use the tuning procedure shown in Figure 64 for tuning *raag*-s in the *kalyan thaat* (roughly analogous to the Lydian mode). The results of this procedure are equivalent to the third mode of the five-limit *shadja grama*.

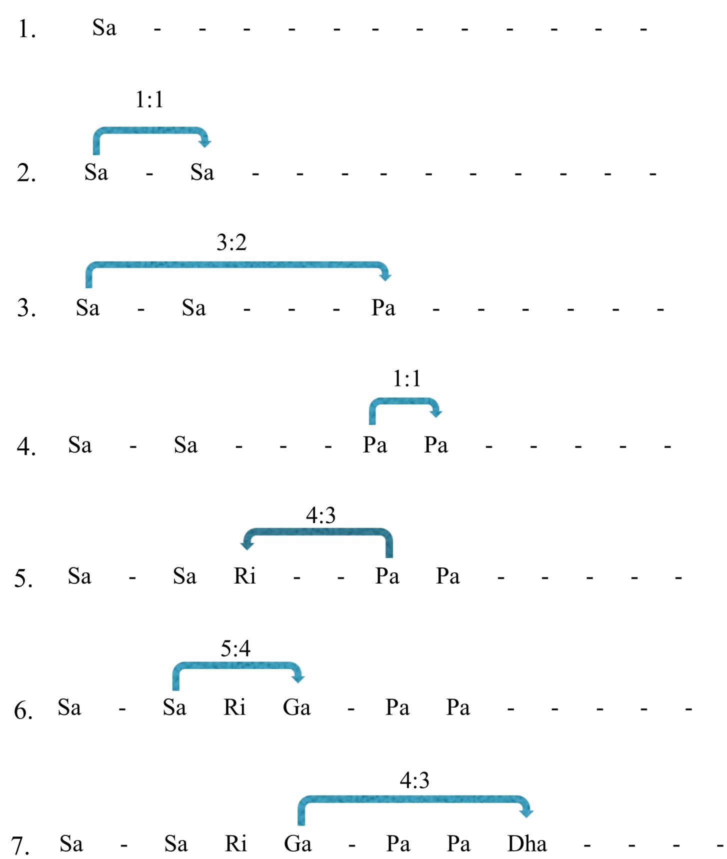

Figure 64. Tuning the third mode of *shadja grama* (a version of *kalyan thaat*) on thirteen sympathetic strings of a sitar.

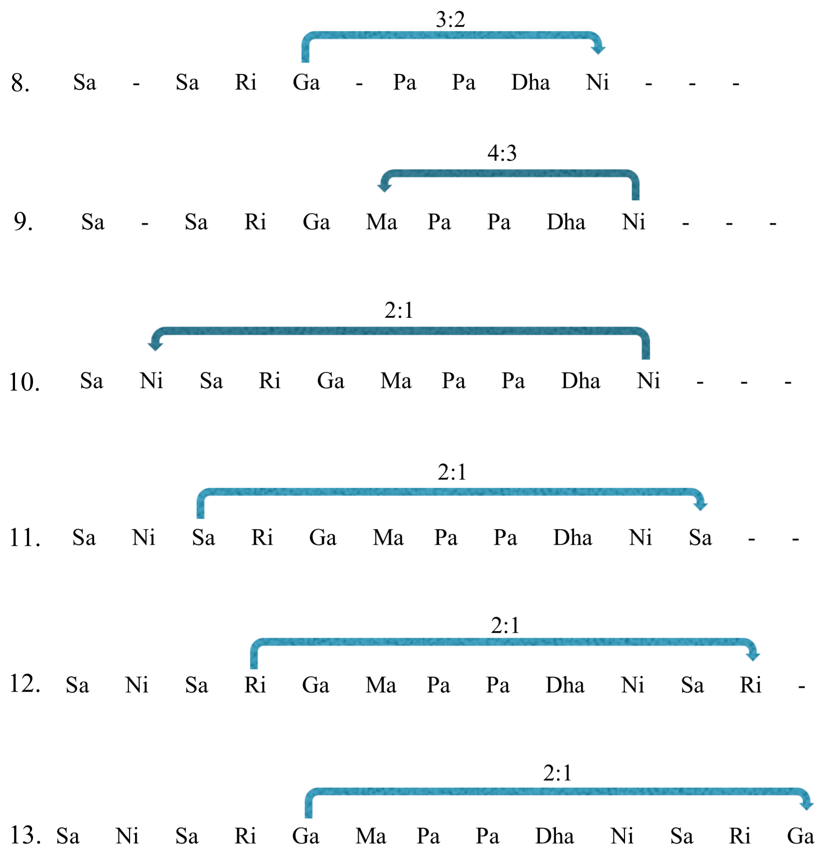

Figure 64 continued.

Conclusion

Music has changed much since Bharata's time, and many commentaries on his tuning system have come and gone. Eventually, music from Persia was brought to India and, as is often the case in the subcontinental melting pot, styles mingled and merged. At some point, the instruments began to reflect an interest in imitating vocal ornaments and glissandi and the harp-*vina* fell out of favor, overtaken by the *rudra vina* and the south Indian lute-style *vina* that can still be heard today. Drastic changes also occurred with the shift to a Sa-based system and the introduction of the harmonically rich and resonant *tanpura*. It is possible that these developments had a significant impact on the direct perception of the microtones and may have expanded the total number of useable *sruti*-s. For example, one could construct a series of very flat, *ati-komal svara*-s by linking fifths and fourths to the seventh harmonic partial, which is often distinctly audible in vibrating *tanpura* strings (Deva 1984). Some of these extremely flat tones sound quite beautiful in relation to an unchanging drone, but are more difficult to incorporate into flexible harp tunings that encourage the type of scalar modulation that Bharata seems to be describing.

Though it is still possible to apply the tuning methods implied in the *Natyashastra* on instruments such as the sitar, recent statements by other authors, researchers, and musicians indicate that, to some, the concept of *sruti* has changed since its inception in *Gandharva* music. In a recent lecture at Rotterdam Conservatory *rudra vina* player Bahauddin Dagar described a series of eighty-four *sruti*-s, with seven assigned to each *svara* (Dagar 2010). Alain Danielou (2010, 32–36) also advocated for a significantly expanded *sruti* count, arriving at sixty-six. In the 1960s, Jairazbhoy and Stone used tape recorders, an oscilloscope, and a film camera to analyze recordings of a handful of artists. Finding no discernable correlation with twenty-two-*sruti* systems, they concluded that "the Indian octave is not composed of twenty-two intervals, but twelve; and that differing intonation . . . is not such a vital part of North Indian classical music today" (Jairazbhoy 1995, 223–44; Jairazbhoy and Stone 1963, 129–31). After running his own experiments in 1976–77, Mark Levy (1982, 85, 136–37) concluded that the tonality of the Hindustani examples he analyzed was best described by "basically twelve semi-tone[s]," which exhibited considerable fluctuation in actual performance. On the other hand, in 2004, Asoke Kumar Datta, Nityananda Dey, and Ranjan Sengupta, conducting research for the ITC Sangeet Research Academy, subjected the recorded performances of 150 songs by 53 artists in 21 different *raag*-s to a pitch extraction algorithm, providing a "non-cognitive approach" to *sruti* determination. They generated a list of twenty-two algorithmically determined values and concluded that "data from individual songs revealed the shruti positions clearly, establishing beyond doubt the use of shrutis in performances of Indian Classical singing" (Datta, Sengupta, and Dey 2011, 1–5; 2006).

The eminent *sarod* player Ali Akbar Khan described his approach to the *sruti*-s in the following passage:

> I am still learning about the microtones. They reach to your heart and help you feel the raags and the notes. In old theory, they say that there are twenty-two in number, but right now I feel that there are twenty-three and a half. There is only one Sa and one Pa. Komal re, ga, and dha all have three. Shuddh ma, tivra ma, shuddh dha, and komal ni each have two. And shuddh re, shuddh ga, and shuddh ni each have one and a half. (Khan and Ruckert 2004, 260)

But he is also quick to suggest that one should "Listen to the music, those masters that you respect. Then you will understand the shrutis. From the books and all those names you will get a wrong idea" (Khan and Ruckert 2004, 260).

Music is, after all, about feeling, personal expression and the experience of beauty. It is through the practice of tuning and playing music that the *Natyashastra* can begin to be understood, not the other way around. Our knowledge of the *sruti*-s enables us to continue exploring music and the aesthetic possibilities of the sonic world, so that we can arrive, just as *Gandharva* musicians did long ago, at our own heart-tuned melodies.

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