In Defense of Music's Eternal Nature:
On the Pre-eminence of **Musica theorica** *Over Musica* **practica**

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by

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Abstract

Since the Renaissance, the normative approach to a philosophy of music has concerned itself primarily with the subjective experience of the listener. This was not always the case. From Greek Antiquity to the Renaissance, music was considered a rigorous, mathematical discipline that shed light on objective truths concerning cosmology and cosmogony. *Musica theorica*, therefore, took precedence over *musica practica* and was taken much more seriously in musical scholarship. Although tension had always existed between *musica theorica* and *musica practica,* such tension reached its peak during the Renaissance and as a result, a shift occurred: *musica theorica* was pushed into the background, and *musica practica* stepped forward.

The intention of this thesis is to convince its reader of the need to bring *musica theorica* back to its proper place in musical and philosophical scholarship, where objective answers can be found, and music's innate eternal nature is revealed. The thesis begins with a historical survey of musical scholarship that eventually brings the reader to the center of the controversy that ensued during the Renaissance, and then forward to present day discussions in philosophy of music that are concerned with music's *subjective and temporal* properties. It is hoped that the reader will see the need for a new shift to occur in philosophy of music that focuses on music's *objective and eternal* properties (that are wholly distinct from the subject experiencing it), and will come away with a new perspective regarding the interdisciplinary nature of philosophy and music.

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“Among the many forms in which the human spirit has tried to express its innermost yearnings and perceptions, music is perhaps the most universal.
It symbolizes the yearnings for harmony with oneself and with others, with nature, and with the spiritual and sacred within us and around us.”*

Dalai Lama

*Press Conference, New Delhi, Monday May 17, 1999, announcing the “World Festival of Sacred Music.”

Chapter 1

An Introduction:
Musica theorica, Musica practica, and Philosophy

Any inquiry involving music seems to necessitate an immediate distinction between *musica theorica* (music theory) and *musica practica* (musical performance)[1]. One possible way of characterizing this distinction is to look upon *musica theorica* as encompassing the *why* of music, and *musica practica* as encompassing the *how*. Philosophy, in general, is also concerned with the *why* of things, yet, any inquiry involving the 'philosophy of music' stays, for the most part, within the boundaries of *musica practica* addressing such questions as : "what kind of emotive response did the musical experience illicit?", or "what kind of meaning was the composer trying to get across to the listener?", or even, "what is it that made that musical experience so beautiful?" When pondering the notion of a 'philosophy of music', one is, for the most part, confined within the sphere of aesthetics. There are no objective truths being sought, and it is the general opinion that there are none to be found.[2] This, however, was not always the case.

Before the Renaissance, music was looked upon as a rigorous discipline, and took its place alongside arithmetic, geometry and astronomy as one of the four branches of the *quadrivium*[3] (the

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[1] This dichotomy is part of a tripartite in musical scholarship which includes *musica poetica* as well. The term *musica poetica* was used in music-theoretical writings; see, for example, Joachim Burmeister, *Musicapoetica* (Tostock: Stephan Myliander, 1606). For a Latin edition and English translation of this treatise, see Benito V. Rivera, translated with introduction and notes, *Joachim Burmeister: Musical Poetics*, part of *Music Theory Translation Series*, ed. by Claude V. Palisca (New Haven, Connecticut and London: Yale University Press, 1993). For a summary of this terminology, see Martin Ruhuke, "Musica theorica, practica and poetica," in *Die Musik in Geschichte und Gegenwart,* 14 vols., ed. by Friedrich Blume (Kassel: Bärenreiter, 1949-1973), Vol. 9 (1961), cols. 949-958.

[2] Note the contributions of such contemporary scholars as Peter Kivy, Harold Fiske, Jean-Jacques Nattiez, and Francis Sparshott which are discussed later in Chapter 4 of this thesis.

[3] See Claude V. Palisca, "Theory, theorists," in: *The New Grove Dictionary of Music and Musicians*, 29 vols.,ed. by Stanley Sadie, John Tyrrell, (London: Macmillan, 2001), Vol. 25, pp. 361-363; Boethius, *Fundamentals of Music*, trans. by Calvin Bower and ed. by Claude V. Palisca as part of *Music Theory Translation Series*, ed. by Claude V. Palisca

cornerstone of Platonic scientific education revived by Boethius in the Middle Ages)[4]. Because of its sublime harmonic perfection and its intrinsic mathematical nature, the principles governing music were thought to be the same as those governing our universe; it was hoped, therefore, that music would shed light on many physical and philosophical problems pertaining to cosmology and cosmogony. From the Pythagoreans of the sixth century B.C.E. to the Florentine Camerata in sixteenth-century Italy, music was often found at the center of ongoing philosophical debates.[5] Although the tension between advocates of *musica theorica* and those of *musica practica* can be found in scholarly works as early as the fourth century B.C.E., it reached its peak in the Renaissance when *musica theorica* came under much scrutiny as the pinnacle of musical study. Boethius' *De institutione musica*, the most authoritative source in the training of musicians (and philosophers) throughout the Middle Ages and well into the Renaissance[6], was eventually disregarded as an obsolete text of little use to the practicing musician. Musical scholars[7] began questioning whether the numerical ratios expounded upon in musical treatises up to this point were worth investigating.

(New Haven, Connecticut and London: Yale University Press, 1989), p. xix; see also Chapter 3 of this thesis.

[4]The *quadrivium* (arithmetic, geometry, astronomy and harmonics) along with the *trivium* (grammar, rhetoric and logic) comprised the seven branches of the *artes liberales*. For a more comprehensive discussion on this subject, and an explication regarding the significance of *artes liberales* in the university education system of the Middle Ages, see Jürgen Sarnowsky, "Die *artes* im Lehrplan der Universitäten," in: *Artes im Mittelalter*, ed. by Ursula Schaefer (Berlin: Akademie Verlag, 1999), pp. 13-33; Guy Beajouan, "L'enseignement du quadrivium," in: *Settimane di studio del Centro italiano di studi sull'alto Medievo* 19 (1972), pp. 639-667; Friedmar Kühnert, *Allgemeinbildung und Fachbildung in der Antike*, Vol. 30 of *Deutsche Akademie der Wissenschaften zu Berlin: Schriften der Sektion für Altertumswissenschaft* (Berlin: Akademie-Verlag, 1961).

[5]See Claude V. Palisca, *Florentine Camerata: Documentary Studies and Translations,* as part of the *Music Theory Translation Series* ed. by Claude V. Palisca (New Haven, Connecticut: Yale University Press, 1989).

[6] See Boethius's *Fundamentals of Music*, trans. by Calvin Bower; Claude V. Palisca, "Boethius in the Renaissance" in *Music Theory and Its Sources*: *Antiquity and the Middle Ages*, Vol. 1 of *Notre Dame Conferences in Medieval Studies,* ed. by André Barbera (Notre Dame, Indiana: University of Notre Dame Press, 1990), pp.259-280.

[7] The contributions of Vincenzo Galilei (late sixteenth century) and René Descartes (early seventeenth century) are discussed in Chapter 3 of this thesis.

The discrepancy between the musician's ear and purely theoretical tuning systems[8] was brought to the foreground and favour fell on the practicing musician. The tone was set and *musica theorica* fell by the wayside, while *musica practica* found its place at center stage.

As the gap widened between these two approaches, the nature of philosophical discussions regarding music changed. Philosophy of music was no longer seeking out objective answers because music was no longer viewed as an objective, scientific discipline. The *why* in music began taking its form in questions such as "why do I feel this way when I listen to this piece of music?" as opposed to "why is it that consonant intervals adhere to consistent numerical ratios?". The former question requires an answer that necessarily involves the subject experiencing the music. The latter question, however, does not involve the subject at all, only the music, in and of itself, or, music *qua* music. It is time to turn back to *music* itself – its mathematical nature and the principle of *harmonia* – in order to understand music for what it is, namely, a manifestation of eternal principles that remain unchanged. It is time to return *musica theorica* to its proper place in musical and philosophical scholarship, where objective answers can be found, and music's innate universal nature is irrefutably revealed.

This thesis begins with the Pythagoreans and the non-Pythagoreans in Greek Antiquity, where tension between *musica theorica* and *musica practica* is evident, but music remains a discipline understood first and foremost by its mathematical nature. Chapter 2 continues the discussion through the Middle Ages, focusing primarily on Boethius' *De institutione musica*, and then moves on to the Renaissance, where the tension between *musica theorica* and *musica practica*

[8] See Mark Lindley, "Temperaments" in: *The New Grove Dictionary of Music and Musicians*, 29 vols. ed. by Stanley Sadie and John Tyrrell, (London: Macmillan, 2001), Vol. 25, pp. 248-268.

culminates in an ongoing controversy among musical scholars[9] that eventually does irreparable damage to music's authoritative place among mathematics and science. Chapter 3 offers several contemporary theories in philosophy and music (such as Harold Fiske's theory of emotion and music and Jean-Jacques Nattiez's theory of semiology) that unveil a firm 'post-Renaissance' conviction that a theory of music must focus entirely on the subject experiencing the music, and not the music itself. The weaknesses that render these theories questionable will be made apparent, as will the need to shift focus to the eternal and objective properties of music that have been neglected for so long.

It is hoped that the reader will come away with a new, or renewed, respect for the interdisciplinary nature of philosophy and music, and the realization that a philosophy of music *can* reveal the objective and eternal nature of music *qua* music.

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[9] This is especially evident through correspondences; see Claude V. Palisca, *Girolamo Mei: Letters on Ancient and Modern Music to Vinzenzo Galilei and Giovanni Bardi*, Vol. 3 of *Musicological Studies and Documents,* ed. by Armen Carapetyan (American Institute of Musicology, 1960); Anthony Kenny, trans. and ed., *Descartes: Philosophical Letters* (Oxford: Clarendon Press), 1970.

Chapter 2

A Return to Greek Antiquity: the Pythagoreans and Non-Pythagoreans

The term *harmonia* brings to mind agreement, concordance, balance and proportion. It is thought to be the opposite of chaos, bringing together autonomous and often opposing elements, and melding them into a melodious consonance: the duality of limited and unlimited, matter and form, temporal and eternal. The concept of *harmonia* is central in classical Greek thought; ἁρμονία meaning literally to 'fit together'. From the fitting together of two pieces of wood to the underlying principle in Greek cosmogony and cosmology, *harmonia* permeates every aspect of being – and from the beginning it has been associated with music[1].

Harmonia is understood as the unifying principle that blends two distinct elements into a single entity. Music manifests the very essence of this unity in sensible pleasure. The pleasant interval that results from two pitches, high and low, sounding simultaneously is in fact the result of perfect and invariable mathematical principles.[2] This is what distinguishes music from noise[3].

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[1] See Thomas J. Mathiesen, "Harmonia and Ethos in Ancient Greek Music," (part of Round Table: "The Ancient Greek Harmoniai, Tonoi and Octave Species in Theory and Practice," organized and chaired by Claude V. Palisca at the National Meeting of the American Musicological Society in Louisville, Kentucky, October 27, 1983), in: *The Journal of Musicology: a Quarterly Review of Music History, Criticism, Analysis and Performance Practice* 3 (1984), pp. 242-251.

[2] What modern scholars know of ancient Greek music theory implies that music was predominantly monophonic (two pitches sounding one after the other in a linear fashion). However, 'concords' (συμφωνίαι) were understood theoretically as two distinct pitches blended, or unified into a single element. See Andrew Barker, "Ptolemy's, Pythagoreans', Archytas's, and Plato's conception of Mathematics," in: *Phronesis* 39 (1994) pp. 113-135, especially Footnote 5. Two more excellent sources are Thomas J. Mathiesen, *Apollo's Lyre: Greek Music and Music Theory in Antiquity and the Middle Ages*, Vol. 2 of *Publications of the Center for the History of Music Theory and Literature* (Lincoln, Nebraska and London: University of Nebraska Press, 1999); Andrew Barker, ed., *Greek Musical Writings II: Harmonic and Acoustic Theory* (Cambridge and New York: Cambridge University Press, 1989).

[3] Jean-Jacques Nattiez, discussed in Chapter 3 below, holds that "music is whatever people choose to recognize as such, [and] noise is whatever is recognized as disturbing, unpleasant, or both"; see Jean-Jacques Nattiez, *Music and Discourse* (New Jersey: Princeton University Press, 1990), pp. 47-48. However, it should be noted that noise is not only considered in its opposition to sonority, but also electronically as interference in the transference of heat, visually as snow on a television screen, and even rumour, as opposed to fact.

Since mathematical principles that are evident in the natural order of the universe are also demonstrated in what we call music, it follows that any enquiry involving harmonics and acoustics must also involve the study of mathematics. The science of harmonics, however, can be reduced to two distinct approaches in ancient Greece: the first is through the study of numerical ratios, the second is strictly empirical. This chapter will attempt to illustrate the tension between these two approaches as it follows the development of musical thought from the Pythagoreans[4] (who saw music, number and the cosmos as interrelated, each an intrinsic part of the other two) through to the phenomenological approach of Aristoxenus (who places music solely within the sphere of human experience). The first section will focus on the Pythagoreans: Philolaus and Archytas; the second section will focus on the 'non-Pythagoreans': Plato, Aristotle and Aristoxenus.

I The Pythagoreans

The figure of Pythagoras is an elusive one: "The attempts of scholarship to grasp the underlying historical reality keep getting entangled in contradictions; where some think they discern the figure of a world-historical genius, others find little more than empty nothingness."[5] He is mentioned only infrequently by earlier writers (such as Archytas and Philolaus), and all that we know to date is that "early Pythagoreanism was more a set of practices than a body of doctrines."[6] Most of the philosophical treatises that are attributed to him are known to us only through other

[4] Although the basic principles of Ptolemy's mathematical approach to music definitely lie within the Pythagorean camp (while at the same time recognizing the importance of sense perception), his *Harmonica* (second century C.E.) will not be discussed in this chapter, but within the context of Boethius' *De institutione musica* in Chapter 3 of this thesis.

[5] Walter Burkert, *Lore and Science in Ancient Pythagoreanism*, trans. by Edwin L. Minar, Jr. (Cambridge, Massachusetts: Harvard University Press, 1972), p. 1.

[6] Andrew Barker, *Harmonic and Acoustic Theory,* p. 28.

Greek scholars[7] whose biases tend to tamper with the original Pythagorean doctrine. Consequently it is up to present-day scholars to sift through these works in order to find a common thread that can be genuinely ascribed to Pythagoras.

We do know that Pythagoras was born in Samos in the sixth century B.C.E. and that his teachings attracted a group of followers who were willing to embrace a way of life that was defined by a strict set of morally based prohibitions and rituals. These prohibitions and rituals were thought to be necessary in achieving purification of the soul, the primary goal of the Pythagorean[8]. The purpose of this purification was to assist in the assimilation of the soul to the ordered unity of the universe – a unity that consists of a system of parts that are held together in harmonious collaboration. *Harmonia*, therefore, is at the heart of Pythagorean inquiry and at the heart of *harmonia* is number.

Arithmetic was the cornerstone of Pythagorean metaphysics; numbers were deemed by them as fundamental constituents of reality.[9] Because the primary intervallic relations in music can be expressed as numerical ratios, music was seen by the Pythagoreans as supervening upon every

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[7] See, for example, Porphyry, *Life of Pythagoras* and Iamblichus' *Vita Pythagorica*, which sought to present the life of Pythagoras in ten volumes, as ten was considered by Pythagoreans to be the perfect number. For further readings in 'source problems', see Walter Burkert, *Lore and Science*, pp. 97-109.

[8] William Jordan writes: "It must be remembered that Pythagoreanism was not a specialized science, like modern physics, for it included elements of mysticism, political theory and ethics. Furthermore, the underlying purpose of the mathematical proofs of Pythagoreanism appears to have been more theological than scientific; those who adhered to the Pythagorean views meditated on these proofs in order to develop spiritual awareness"; William Jordan, "Galileo and the Demise of Pythagoreanism," in: *Music and Science in the Age of Galileo*, ed. by Victor Coelho as Vol. 51 of *The University of Western Ontario Series in Philosophy of Science*, ed. by Robert E. Butts (Dordrecht, Netherlands and Boston, Massachusetts; and London: Kuwer Academic Publishers, 1992), pp. 45-63.

[9] For further reading on this subject, see André Barbera, "The Persistence of Pythagorean Mathematics in Ancient Musical Thought," (Unpublished Ph. D. Dissertation, University of North Carolina at Chapel Hill, 1980); Barbara Münxelhaus, *Pythagoras musicus: Zur Rezeption der pythagoreischen Musiktheorie als quadrivialer Wissenschaft im lateinischen Mittelalter*, Vol. 19 of *Orpheus-Schriftenreihe zu Grundfragen der Musik*, ed. by Martin Vogel (Bonn-Bad Godesberg: Verlag für Systematische Musikwissenschaft, 1976).

aspect of cosmogony and cosmology [10]. Consequently, Pythagoras gave much theoretical attention to those numerical ratios that can be observed from a vibrating string.

If one plucks a taut length of string, a pitch will sound. If that string is divided into two, and one half of the string is plucked, then a pitch will sound that is identical but in a higher register, as the string is vibrating exactly twice as fast. The unified relation of two identical pitches, one sounding high and the other low, provides us with the most consonant of intervals, the *diapason* (octave). The diapason can be expressed as the ratio 2:1 and is made up of two lesser consonant intervals: the *diapente* (fifth - 3:2) and the *diatessaron* (fourth - 4:3). The four numbers that comprise these three fundamental ratios of *harmonia* form the *tetractys* (τετραχτύς) of the decad (*Figure 1*), the first principle of Pythagorean cosmology[11] : "and it may well have seemed the

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[10] Despite Pythagoreanism being pushed into the background with seventeenth- century rationalism, and the fact that Pythagoras' theory of proportion with regard to the planets has been proven wrong, the ideology behind the 'harmony of the spheres' that permeated antiquity continues to intrigue scholars. This is evident in such works as Johannes Kepler's *Harmonices mundi* (1619), Robert Fludd's *Utriusque cosmi* (1617) and Marin Mersenne *(Harmonie universelle* (1636)), and contemporary works such as James Haar, *Musica mundana: Variations on a Pythagorean Theme,* (Unpublished Ph.D. Dissertation, Harvard University, 1960); Marius Schneider, "Die musikalischen Grundlagen der Sphärenharmonie," in: *Acta Musicologica* 32 (1960), pp. 136-151; Peter Bicknell, "Early Greek Knowledge of the Planets," in: *Eranos: Acta Philologica Suecana* 68 (1970), pp. 47-54; Hans Schavernoch, *Die Harmonie der Sphären: Die Geschichte der Idee des Welteinklangs und der Seelenstimmung,* Vol.6 (Special Issue) of *Orbis Academicus: Problemgeschichten der Wissenschaft in Dokumenten und Darstellungen* (Freiburg im Breisgau and Munich: Karl Alber, 1981); Friedrich Zipp, *Vom Urklang zur Weltharmonie: Werken and Wirken der Idee der Sphärenharmonie* (Berlin and Kassel: Merseburger, 1985); Jamie James, *The Music of the Spheres: Music, Science and the Natural Order of the Universe* (New York: Grove Press, 1993).

[11] Put simply, the decad, or *ten*, is made up of *one* plus *two* plus *three* plus *four*. Pythagoras has been cited as saying, "What you suppose is four is really ten, and a perfect triangle, and our Oath"; see Walter Burckert, *Lore and Science,* p.72 (Footnote 120). Further, the tetractys represents point, line, plane and solid. Geometry, that takes its place alongside music in the *quadrivium*, is derived from it: "Numbers gave rise to geometric figures: a second unit placed alongside the first generated a line from a point, a third unit placed so as to form a triangle produced the basic two-dimensional figure, and a fourth unit placed on top of these made a tetrahedron, the first three-dimensional figure"; see Edward A. Lippman, *Musical Thought in Ancient Greece* (New York: Da Capo Press, 1975 is a reprint of New York: Columbia University Press, 1964), p. 9. For a more comprehensive discussion on the tetractys, see Theo Reise, *Das Geheimnis der pythagoreischen Tetraktys,* Vol. 3 of *Harmonikale Studien* (Heidelberg: Lambert Schneider, 1967); for evidence of the tetractys in art history, see Julius Schwabe, "Hans Kaysers letzte Entdeckung: Die pythagroeische Tetraktys auf Raffaels "Schule von Athen," in: *Symbolon: Jahrbuch für Symbolforschung* 5 (1966), pp. 92-102; for a classification of the Greek terms used here and below, see Solon Michaelides, *The Music of Ancient Greece: An Encyclopedia* (London: Faber and Faber, 1978).

supreme manifestation of the mysterious power of the *tetractys* that the same first four numbers express the basic ratios of the musical intervals."[12]

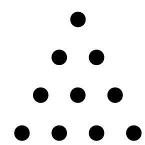
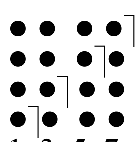


Figure 1

The underlying principles of Pythagorean cosmogony are the paired contraries, the limited and the unlimited and all numbers are seen to be generated by them.[13] Odd is seen as manifesting the principle of the limited (which is stable) and even is seen as manifesting the principle of the unlimited (which is changing). As we can see from the gnōmōn (literally a carpenter's rule – another symbolic figure offered by Pythagoras), all number is generated from the limited and the unlimited: odd is limited and square *(Figure 2)*, even is unlimited and oblong *(Figure 3)*.

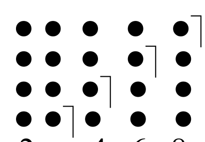
The gnōmōn (γνώμων)



1, 3, 5, 7, etc.

odd, limited, square, stable

Figure 2



2, 4, 6, 8, etc.

even, unlimited, oblong, changing

Figure 3

[12] Walter Burkert, *Lore and Science,* p. 478.

[13] Edward A. Lippman, *Musical Thought in Ancient Greece*, pp. 9-10.

Philolaus[14], the first known Pythagorean to write an exposition of the Pythagorean system,[15] argues that all things in the universe, including the soul, belong to distinct and contrary categories – the limiting and the unlimited (περαίνοντα και ἄπειρα). The limiting and the unlimited are wholly distinct (associated with odd (περισσόν) and even (ἄρτιον), respectively) and irreconcilable without the third principle, the 'One', also known as *harmonia* (ἁρμονία): "*Harmonia* in every way arises out of opposites. For *harmonia* is the unification of what is a mixture of many ingredients, and the agreement of the disagreeing."[16]

Before discussing the writings of Philolaus, or any of the early Pythagoreans, one must raise the problem of authenticity, for there is a great deal of writing falsely attributed to Pythagoras and his pupils[17]. Even in ancient times there was much debate over whether Pythagoras had written anything at all: "The opinion was widespread that no such book had been preserved, or even that Pythagoras avoided the written word on principle and recorded his teachings only in the minds of his disciples."[18] As for Philolaus and Archytas, two renowned Pythagoreans, not much is known for

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[14] Philolaus, born *c*. 470 BC, was a contemporary of Socrates. An excellent source and commentary on the life and writings of Philolaus is Carl A. Huffman, *Philolaus of Croton: Pythagorean and Presocratic* (Cambridge: Cambridge University Press, 1993).

[15] Sir Thomas Heath, trans., *Aristarchus of Samos: The Ancient Copernicus*. (Oxford: Clarendon Press, 1959 as reprint of first edition, 1913) p. 47.

[16] Fragment 10, attributed to Philolaus, as cited in Carl A. Huffman, *Philolaus of Croton*, p. 416. See also Hermann Dies, *Die Fragmente der Vorsokratiker,* 3 vols., ed. by Walther Kranz (Berlin-Grünewald: Weidmannsche Verlagsbuchhandlung, 1951-1952).

[17] Such as Lasus of Hermione(*ca*. 510 B.C.E.), Hippasus of Metapontum (*ca*. 500 B.C.E.), Philolaus (*ca*. 470-385 B.C.E.), Archytas of Tarentum (*ca*. 428-350 B.C.E.), Nicomachus (*ca*. 60-120 C.E.) and Aristides Quintilianus (*ca*. third-century C.E.). See Andrew Barker, *Harmonic and Acoustic Theory,* pp. 30-45 and Walter Burkert, *Lore and Science,* Section II, pp. 97-217.

[18] Walter Burkert, *Lore and Science,* p. 218.

certain.[19] Fragments attributed to Philolaus include many that are likely unauthentic,[20] and much of the written work that we have of Archytas is apocryphal.[21] Despite this difficulty, it is worthwhile to sift through the spurious bits to find the gems offered by these two influential Pythagoreans.[22]

Philolaus' understanding of our world rests on the principle of *harmonia* (ἁρμονία). In the most definitive sense, *harmonia* means 'fitting together', which is suitable if one is to posit a principle that unites the distinct and separate constituents of a whole: "Nature in the world-order was fitted together both out of things which are unlimited and out of things which are limiting, both the world-order as a whole and all things in it."[23] There is no doubt, however, that Philolaus utilizes this principle, first and foremost, in its musical sense.

Philolaus does not make any deep speculative inquiry into *how* Being (ἐστώ) comes into existence, or its unifying principle, *harmonia*. The world exists, and it is ordered. What Philolaus investigates are relationships, and he finds that our world is ordered according to number. Philolaus' presuppositions begin with Being in general, then the opposition of limiting and unlimited, and finally *harmonia*, which is expressed as a numerical relationship. Philolaus writes:

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[19] They are, however, cited in many scholarly works throughout the Middle Ages and into the Renaissance, such as Franchino Gaffurio's *Theorica Musice* (1492), see Clement A. Miller, trans. *Franchinus Gaffurius*: *De harmonia musicorum instrumentorum opus,* Vol. 33 of *Musicological Studies and Documents*, ed. by Armen Carapetyan (American Institute of Musicology, 1977). Further, according to a source attributed to Cicero (Cic. *De oratore.* 3.139), Philolaus is referred to as the teacher of Archytas; see Walter Burkert, *Lore and Science,* p. 228 (Footnote 50).

[20] Andrew Barker, *Harmonic and Acoustic Theory*, p. 36; Carl A. Huffman, *Philolaus of Croton,* pp. 17-37; Walter Burkert, *Lore and Science,* pp. 238-276.

[21] Walter Burkert, *Lore and Science,* p. 222.

[22] For a more in depth discussion on authenticity, see Walter Burkert, *Lore and Science,* pp.97-109 and Carl A. Huffman, *Philolaus of Croton,* pp. 17-35.

[23] Philolaus, Fr. I, as cited in Carl A. Huffman, *Philolaus of Croton,* p. 93.

> The magnitude of *harmonia* (fitting together) is the fourth [diatessaron] and the fifth [diapente]...the fifth is greater than the fourth by the ratio 9:8 [a tone]. For from *hypat*ē [ὑπάτη, or lowest tone] to the middle string *mes*ē (μέση) is a fourth, and from the middle string to *neat*ē [νεάτη, or highest tone] is a fifth, but from *neat*ē to the third string is a fourth, and from the third string to *hypat*ē is a fifth. That which is in between the third string and the middle string is the ratio 9:8 [a tone], the fourth (diatessaron) has the ratio 4:3, the fifth (diapente) 3:2, and the octave (diapason) 2:1. Thus the *harmonia* is five 9:8 ratios [tones] and two *diesis* [δίεσις, or smaller semitones]. The fifth is three 9:8 ratios [tones] and a *diesis*, and the fourth two 9:8 ratios [tones] and a *diesis*.[24]

In this fragment, *harmonia* is considered in its musical sense [25] – as a scale demarcated by the octave (diapason) – but the essence of musical *harmonia* is the same as that *harmonia* which binds the limiter and the unlimited into the world as we know it.

For Philolaus, any philosophical or epistemological ideas are merely elucidating what exists already, namely a world that consists of a pair of basic opposites, instantiated by the principle of *harmonia*[26] and delimited by number. He explicitly refuses to make any claims about Being *qua* Being (in contrast to Aristotle, which is discussed later). His aim is to investigate the many relationships of number. To reiterate, Philolaus' cosmos is made up of limiting and unlimited (περαίνοντα καὶ ἄπειρα) and the unifying principle *harmonia* (ἁρμονία). He parallels this notion with 'odd',' even', and 'the One': "Number indeed has two proper kinds, odd (περισσόν) and even (ἄρτιον), and a third from both mixed together, the even-odd (ἀρτιοπέριττον). Of each of the two kinds there are many forms, of which each thing itself gives signs."[27] Just as the opposing

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[24] Fragment 6, as cited in Carl A. Huffman, *Philolaus of Croton,* pp. 146-147.

[25] This is discussed in further detail below in this chapter.

[26] Note Aristotle, *On the Universe*, 396b25 - 397a5 as found in: Clement A. Miller, trans. *Franchinus Gaffurius*: *De harmonia.* Further evidence can be found in a source attributed to Cicero (Cic. *De oratore* 3.139).

[27] Fragment 5, as cited in Carl A. Huffman, *Philolaus of Croton,* p. 178.

principle of limiting and unlimited is transcended by the synthesizing principle of *harmonia*, so the opposing principle of odd and even is transcended by the synthesizing principle of the 'even-odd ', or the One. Walter Burkert writes: "Without any doubt...the odd numbers are the limiting (περαίνοντα) and the even the unlimited (ἄπειρα). The ἀρτιοπέριττον, made from the mixture of the two, is the πρᾶτον ἁρμοσέν, that is, the One."[28] Fragment five is particularly significant, as it is the only fragment of Philolaus that makes specific reference to number itself, and its relationship to the world. All things contain number, and insofar as they consist of περαίνοντα they contain the odd, ἄπειρα they contain the even, and both are bound together in *harmonia*.[29] Similarly, the even and the odd numbers that define the ratios intrinsic to musical intervals and the diatonic scale, are bound together in *harmonia*.

The entire octave length scale can be described in terms of numerical ratios, and it is Philolaus who provides us with the earliest written account of such a description.[30] The most fundamental intervals in music (as mentioned above) are that of the diapason (2:1), the diatessaron

[28] Walter Burkert, *Lore and Science,* p. 264.

[29] Philolaus posits *harmonia* as the binding principle of all that exists. There is nothing in his fragments, however, indicating that he said anything about the harmony of the spheres. Pythagoras held that the universe is constructed in such a way that the motion of the seven heavenly bodies creates a perfect harmony, each planet sounding a note that corresponds to the notes of the Heptachord (a scale comprised of seven pitches that are tuned to the diatonic scale – fundamental in early Greek music). This would have caused problems for Philolaus, for according to his fragments, the Pythagorean system is comprised of *ten* heavenly bodies which would have to correspond to ten notes (see Sir Thomas Heath, trans., *Aristarchus of Samos,* Chpt. 12). For a further explication on the symbolism pertaining to ancient Greek number symbolism and the number seven, seeWilhelm Heinrich Roscher, *Über Alter, Ursprung und Bedeutung der hippokratischen Schrift von der Siebenzahl: Ein Beitrag zur Geschichte der ältesten griechischen Philosophie und Prosaliteratur,* Vol. 28/5 of *Abhandlungen der Philosophisch-Historischen Klasse der Königlich – Sächsischen Gesellschaft der Wissenschaften* (Leipzig: B.G. Teubner, 1921); Wilhelm Heinrich Roscher, *Die Sieben – und Neunzahl im Kultus und Mythus der Griechen, nebst einem Anhang: Nachträge zu den "enneadischen und hebdomadischen Fristen und Wochen" enthaltend*, Vol. 24/1 of *Abhandlungen der Philosophisch-Historischen Klasse der Königlich – Sächsischen Gesellschaft der Wissenschaften* (Leipzig: B.G. Teubner, 1904); Martin Vogel, "Die Zahl Sieben in der spekulativen Musiktheorie" (Unpublished Ph.D. Dissertation, Universität Bonn, 1953).

[30] See Philolaus' Fragment 6 cited above in this thesis.

(4:3) and the diapente (3:2). If we go up the interval of a fourth (diatessaron) from any given pitch, and then up the interval of a fifth (diapente), the pitch that we arrive at will be an octave (diapason) above the original pitch.[31] The octave can then be said to be made up of a fourth and a fifth, if the ratios that govern the fourth and the fifth are added by multiplying the terms (3:2 × 4:3 = 12:6 = 2:1). The interval of the whole tone, which is regarded as the basic unit of the scale, is located, respectively, between the pitch found a fourth above the original pitch and a fifth above the original pitch. This interval corresponds to the ratio 9:8, which is the difference of 3:2 and 4:3 (to subtract ratios, one must divide the terms or cross multiply, hence 3:2 ÷ 4:3 = 3:2 × 3:4 = 9:8 [32]). As the fifth can be regarded as a fourth plus the whole tone, so the octave can be regarded as two fourths plus a whole tone. The fourth consists of two whole tones plus a 'remainder', or *diesis* (4:3 - (9:8)(9:8) = 4:3 ÷ 81:64 = 4:3 × 64:81 = 256:243). Philolaus' diatonic scale thereby consists of the following intervals: 9:8, 9:8, 256:243 (the fourth, or *tetrachord*[33]) plus the fifth, 9:8, 9:8, 9:8, 256:243 (in other

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[31] For reasons of clarity, henceforth I will refer to the diatessaron as a 'fourth', the diapente as a 'fifth', and the diapason as the'octave'.

[32] For further explication of 'addition' and 'subtraction' of intervals, see André Barbera, "Pythagorean Scale," in: *The New Harvard Dictionary of Music,* ed. by Don Michael Randel (Cambridge: Harvard University Press, 1986), p. 673.

[33] In Greek harmonic theory (as in music theory today) the tetrachord is considered to be a foundational building block of the musical scale. It is generally divided into three intervals. In Greek music, the intervals are subject to change. Aristoxenus was the first to draw attention to the concept of *genera*. The *genera* is determined through moveable notes within the tetrachord. In other words, the two outer notes of the tetrachord are considered stable in pitch, while the two middle notes are moveable, depending on what *shade* of music one wants to produce. Aristoxenus labeled the three *genera* of the tetrachord: the *diatonic,* the *chromatic* and the *enharmonic* ('the oldest and finest'). The interval size within the parameters of the fixed notes differed according to which *genera* was being expressed, and consequently, the numerical ratios expressing those intervals would differ accordingly. See Annie Bélis, "Aristoxenus," in: *Dictionary of Music and Musicians*, 29 vols., ed. by Stanley Sadie and John Tyrrell (London: Macmillan, 2001), Vol.2, p. 2 and also Archytas, below in Chapter 2 of this thesis. For further reading on the tetrachord, see André Barbera, "Arithmetic and Geometric Divisions of the Tetrachord," in: *Journal of Music Theory* 21 (1977), pp. 29-42.

words, a whole tone plus the fourth).[34] This fundamental division of the scale that Philolaus has revealed (and what we understand today as the major scale: whole-tone, whole-tone, semitone, whole-tone, whole-tone, whole-tone, semitone) makes perfect mathematical sense.[35]

Archytas of Tarentum – a Greek mathematician and philosopher, and a contemporary of Plato – took these fundamental arithmetic principles of Pythagorean harmonic theory a step further.[36] First, he recognized that pitch is dependent upon the speed with which the sound is made and travels: "Now when things strike against our organ of perception, those that come swiftly and powerfully from the impacts appear high-pitched, while those that come slowly and weakly seem to be low-pitched."[37] Archytas was only slightly misled, for he was correct in his assumption that pitch is somehow determined by the speed of vibrations. It is not, however, the speed with which sound travels from the object to the recipient that determines the pitch (all sounds travel at an equal velocity given the same medium), but the number of sound waves within a measurement of time.[38] Second, he made a serious attempt at reconciling the tension between the strictly mathematical

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[34] Plato adopts this scale as an intrinsic part of the world soul in the *Timeaus* (36a-b) which will be further explained below.

[35] Again, as expounded upon in Footnote 33 above, there were different species of scale in Greek music theory (the diatonic, the chromatic, and the enharmonic). These scales will be discussed briefly below in this thesis with Archytas.

[36] Although the majority of work attributed to Archytas is regarded as spurious, his treatise *Harmonics* is among those considered to be authentic.

[37] Archytas Fragment 1, as cited in Andrew Barker, *Harmonic and Acoustic Theory,* p. 40.

[38] The study of sound waves does well to support the Pythagorean approach to music – if one is to measure and chart the sound waves of the perfect fifth, perfect fourth and the perfect octave, one will discover that a distinct pattern repeats itself. When the same is done with dissonant intervals such as a major second, there is no distinct pattern. The study of sound waves was revived in the Renaissance under Giovanni Battista Benedetti (1530-1590) who investigated the mechanics underlying the production of consonances which he discusses in works such as *Diversarum speculationum mathematicarum & physicorum liber* (1585) and in two undated letters addressed to Cipriano de Rore (1653); see Claude V. Palisca, "Scientific Empiricism in Musical Thought," in *Seventeenth Century Science and the Arts*, ed. by Hedley Howell Rhys (Princeton, New Jersey: Princeton University Press, 1961), pp. 91-137; and Chapter 3 below in this thesis.

approach to music, and the strictly empirical one. For he recognized both the Philolaic diatonic scale (usually referred to as the Pythagorean diatonic) which is based on pure mathematical abstraction, *and* the fine tuning of the musician's ear. Archytas' division of the octave deviates from Philolaus', but this is not a result of questionable mathematical theory on either part. Using the Pythagorean proportions of the scale as a starting point, Archytas created a mathematically rigorous system of proportions that allows for the altered tuning of the musician.[39]

Where Philolaus and his predecessors had devised a system that accounts for the fundamental concords within the diatonic scale, Archytas devised a more thorough system that includes the intervals within the tetrachord. He believed not only that the tetrachord could be divided in such a way that would reveal commensurate relations between the intervals, but that the ratio of such intervals would be 'epimoric' (or superparticular).[40]

Archytas devised three different types of scales to depict three different *genera*, or *shades*: the enharmonic, the chromatic, and the diatonic. The tetrachords that these respective scales are built upon contain different interval sizes, and therefore different numerical ratios[41], yet all of the intervals 'add up' to the ratio that is the perfect octave:

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[39] Although Ptolemy (*ca*. 87-150 B.C.E.) criticizes Archytas' division of the diatessaran (the fourth) and diapason (the octave), their approach is similar in that they both take the musician's ear into consideration. Ptolemy is discussed in Chapter 3 below within the context of Boethius' treatise on music.

[40] **n+1: n**; see Andrew Barker, *Harmonic and Acoustic Theory*, p.43 (Footnote 63).

[41] The modern major scale that we are familiar with adheres closely to Philolaus' diatonic scale: (**fourth 4:3**) **whole tone 9:8** (**fourth 4:3**) = **octave 2:1**. However, our half-tones have been modified with equal temperament; see Footnote 94 of this chapter and Footnotes 64 and 67 of Chapter 3 below in this thesis. For a more detailed account of these scales, see Andrew Barker*, Harmonic and Acoustic Theory,* pp. 46-52, and Cristiano M.L. Forster, "Greek Classification of Ratios Tetrachords, Scales and Modes, " in: Cristiano M.L. Forster, *Musical Mathematics: Western Tuning Theory and Practice*, Chapter 10, Part II (2004), pp. 445-457. URL= http://www.chrysalis-foundation.org./musical_mathematics.htm.

______*Enharmonic*: (5:4, 36:35, 28:27) **9:8** (5:4, 36:35, 28:27) = **2:1**

(5/4 × 36/35 × 28/27) = (5070/3780) = 4(r.1)/3 or **4 : 3**

therefore: (**fourth**) **whole tone** (**fourth**) = **octave**

Chromatic: (32:27, 243:224, 28:27) **9:8** (32:27, 243:224, 28:27) = **2:1**

(32/27 × 243/224 × 28/27) = (217,726/163,296) = 4(r.2)/3 = **4 : 3**

therefore: (**fourth**) **whole tone** (**fourth**) = **octave**

Diatonic: (9:8, 8:7, 28:27) **9:8** (9:8, 8:7, 28:27) = **2:1**

(9/8 × 8/7 × 28/27) = (2016/1512) = 4/3 = **4 : 3**

therefore: (**fourth**) **whole tone** (**fourth**) = **octave**[42]

In his *Harmonic and Acoustic Theory*, Andrew Barker suggests that Archytas was pursuing two goals: "He attempted both to analyze the attunements underlying contemporary musical practice, and to reveal the principles of mathematical order on which they were based."[43] Consequently, the numerical ratios depicting the intervals within the tetrachords of the *enharmonic* and the *chromatic* scales do not add up exactly to the perfect 4:3 ratio of the fourth. The discrepancy is minimal in both scales, however, and this is acceptable. As Andrew Barker explains, Archytas wanted to reveal the mathematical principles on which these scales are *based.* He noticed that musicians used the Pythagorean diatonic to tune their instruments, and then tweaked the strings a bit to make particular intervals more consonant to the ear.[44] Archytas wanted to demonstrate that

[42] For further reading, see Martin Vogel, "Über die drei Tongeschlechter des Archytas," in: *Bericht über den Internationalen Musikwissenschaftlichen Kongreß Hamburg 1956*, ed. by Walter Gerstenberg *et al.* (Kassel and Basel: Bärenreiter, 1957), pp. 233-235.

[43] Andrew Barker, *Harmonic and Acoustic Theory,* p. 51.

[44] This altered specifically the ditone: the true ditone in the Greek diatonic scale of Archytas' time is 81:64, yet musicians found that diminishing it slightly created a sweeter sound — the interval then becomes what we know as the major third whose ratio is 5:4 — this actually supports Archytas' position that harmonic theory can be can be reduced to a mathematical system of epimoric ratios; see also Ptolemy in Chapter 3 of this thesis.

'sweetening the interval' (and thereby altering the mathematical proportion of the interval) did not conflict with the mathematical theory that revealed the ratios. Hence there is a reconciliation of the strictly mathematical and the strictly empirical.

Archytas began with the Pythagorean diatonic, yet made further divisions by inserting the harmonic and arithmetic means[45] within intervals in hopes of accounting for the discrepancies between the musicians ear and pure numerical proportion. Although he sought to create a mathematically ordered system of epimoric ratios in his harmonic system (as mentioned above), his chromatic scale reveals what appears to be inconsistencies. On the one hand, his chromatic scale may exemplify the fact that "[his] divisions present several features that no purely mathematical principles will explain"[46]; on the other hand, he may have allowed for the discrepancy of 'epimoric' ratios in this particular scale to account for the musician's ear, while still arguing that the musician's natural 'tuning', or disposition of the ear, can be reduced to pure mathematical theory.[47]

II The Non-Pythagoreans

While Archytas attempted to reconcile the empirical level of musical practice with the metaphysical level of pure mathematical theory, Plato turned away from the empirical entirely and he "[rejected the Pythagoreans'] comparative measurements as the most contemptible

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[45] The harmonic mean between two numbers is as follows $\mathbf{a} : \frac{\mathbf{2ab}}{\mathbf{a+b}} : \mathbf{b}$; therefore, the harmonic mean between 1 and 2 is $\frac{4}{3}$

The arithmetic mean between two numbers is as follows: $\mathbf{a} : \frac{\mathbf{a+b}}{\mathbf{2}} : \mathbf{b}$; therefore the arithmetic mean between 1 and 2 is $\frac{3}{2}$.

[46] Andrew Barker, *Harmonic and Acoustic Theory*, p. 49.

[47] It is worthwhile to note that even scholars such as Franchino Gaffurio (1451-1522) who advocated Pythagorean tuning, could see that keyboards were tempered by flattening the fifth, *Theorica musice* (1496), Book 2, Chapter 3, as translated by Walter K. Kreyszig, pp. 60-63; see Claude V. Palisca, "Theory, theorists," p. 373.

empiricism."[48] For Plato, the science of harmonics is supposed to reveal to us the innate nature of truth which transcends perceptual knowledge. He considered music to be a reflection of the construction of reality, not the result of one's imagination or musicianship. It was his view "that logical and/or mathematical entities subsist independently both of the empirical world and of human thought."[49] As a result, he made it very clear that any attempt to develop a mathematical system based on the ear of actual musicians was futile. Although Plato found within Archytas' theory of the harmonic and arithmetic means and proportion an ideal system of harmonics (as is expounded upon at length in his *Timaeus* [50]), he considered the musician's tuning an imperfect reflection of the actual mathematical principles inherent in music's harmonic structure:

> Just as geometry has as its subject matter such intelligible entities as the square and the circle, which are not things that can be perceived, and is not concerned with the description of the individual, perceptible and approximate squares and circles that it uses in its diagrams, so astronomy and harmonics are concerned with an ideal mathematics of motion. The visible movements of the stars and the audible movements that constitute sounds are to be treated merely as 'diagrams' or perceptual aids, from which the mind can be led to a grasp of the intelligible mathematical principles that perceptible movements may imperfectly exhibit.[51]

In keeping with the Pythagorean tradition, Plato treats astronomy and harmonics as interdependent. The inherent mathematical principles of ratio and proportion that are revealed in

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[48] Erich Frank, as cited in Walter Burkert, *Lore and Science,* p. 373.

[49] M. J. Inwood, "Platonism," in: *The Oxford Companion to Philosophy*, ed. by Ted Honderich (Oxford: Oxford University Press, 1995), p. 686.

[50] A good thorough source is *The Collected Dialogues of Plato, Including the Letters*, ed. by Edith Hamilton and Huntington Cairns (New York: Pantheon Books, 1961).

[51] Andrew Barker on Plato, as cited in: Andrew Barker, *Harmonic and Acoustic Theory,* p. 53.

the science of harmonics are thought to be the same that govern motion in the heavens: "The eyes are made for astronomy,' I [Socrates] said, "and by the same token the ears are presumably made for the type of movement that constitutes music. If so, these branches of knowledge are allied to each other. This is what the Pythagoreans claim, and we should agree..."[52] However, as mentioned above, Plato deviates from the Pythagoreans, chastising them for engaging in an empirical line of inquiry– "They laboriously measure the interrelations between audible concords and sounds, which is as useless an activity as anything astronomers get up to...They limit their research to the numbers they can find within audible concords, but they fail to come up with general matters for elucidation, such as which numbers form concords together and which do not, and why some do and some do not."[53] Pursuing a time consuming study of the mere 'shadows of truth' seems pointless to Plato.

He does concede, however, that "...the body of the cosmos was harmonized by proportion and brought into existence"[54] and he adopts much from Philolaus and Archytas[55]. As is evident from the passage below[56], his construction of the "World Soul"can be directly translated into a musical scale:

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[52] *Republic* 530d; *The Collected Dialogues of Plato*, ed. by Edith Hamilton and Huntington Cairns.

[53] *Republic,* 531a - 531d; *The Collected Dialogues of Plato*, ed. by Edith Hamilton and Huntington Cairns.

[54] *Timeaus* 32a-b; *The Collected Dialogues of Plato*, ed. by Edith Hamilton and Huntington Cairns.

[55] The influence of Philolaus and Archytas becomes especially apparent upon closer inspection of the *Timeaus,* as Plato utilizes Philolaus' diatonic scale, and Archytas' arithmetic and harmonic mean (see below in this chapter). Also, Carl A. Huffman claims: "It is common to refer to the Pythagorean influence of Plato, but the Pythagorean named is usually Plato's contemporary Archytas, who is mentioned in the seventh letter although not in any of the dialogues. Yet, there are persistent reports that make a connection between Plato and Philolaus. A report by Hermippus...has Plato himself buying a book from Philolaus' relatives or else given the book as a reward for saving one of Philolaus' students from Dionysius, the tyrant of Syracuse. Plato is said to have transcribed his *Timeaus* from this book which probably is to be understood as written by Philolaus"; see Carl A. Huffman, *Philolaus of Croton*, pp. 4-5.

[56] *Timeaus,* 35a - 36c; *The Collected Dialogues of Plato*, ed. by Edith Hamilton and Huntington Cairns.

Whereas he made the soul in origin and excellence prior to and older than the body, to be the ruler and mistress, of whom the body was to be the subject. And he made her out of the following elements and on this wise. From the being which is indivisible and unchangeable, and from that kind of being which is distributed among bodies, he compounded a third and intermediate kind of being [οὐσίας]. He did likewise with the Same [αὖ πέρι] and the Different [θατέρου][57], blending together the indivisible kind of each with that which is portioned out in bodies. Then, taking the three new elements, he mingled them all into one form, compressing by force the reluctant and unsociable nature of the Different into the Same. When he had mingled them with the intermediate kind of being and out of three made one, he again divided this whole into as many portions as was fitting, each portion being a compound of the Same, the Different, and Being. And he proceeded to divide after this manner. First of all, he took away one part of the whole [1], and then he separated a second part which was double the first [2], and then he took away a third part which was half as much again as the second and three times as much as the first [3], and then he took a fourth part which was twice as much as the second [4], and a fifth part which was three times the third [9], and a sixth part which was eight times the first [8], and a seventh part which was twenty-seven times the first [27].[58] After this he filled up the double intervals [that is, between 1, 2, 4, 8] and the triple [that is, between 1, 3, 9, 27] [59], cutting off yet other portions from the mixture and placing them in the intervals, so that in each interval there were two kinds of means, the one exceeding and exceeded by equal parts of its extremes [as for example, 1, 4/3, 2, in which the mean 4/3 is one third of 1 more than 1, and one third of two less than two], the other being that kind of mean which exceeds and is exceeded by an equal number. [60] Where there were intervals of 3/2 [hemiolic], and of 4/3 [epitritic], and of 9/8 [epogdoic] made by the connecting terms in the former intervals[61], and he filled up all the intervals of 4/3 [epitritics] with the

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[57] Note the similarities between Plato's 'Sameness" and "Difference" and Philolaus' "Limiting" and "Unlimited", as recorded in Philolaus, Fragment 1, cited in Carl A. Huffman, *Philolaus of Croton,* p. 93.

[58] These numbers (in square parenthesis) are arranged in two branches as follows, the left side displaying 2, 2^2, 2^3; the right side displaying 3, 3^2, 3^3:

1
2 3
4 9
8 27

[59] See p. 23 below in Chapter 2 of this thesis.

[60] The arithmetic and harmonic means (that are borrowed directly from Archytus) are explained in Footnote 45 above; see also Thomas J. Mathiesen, *Appollo's Lyre*, p. 427.

[61] A hemiolic interval (3:2) is the musical fifth; an epitritic interval (4:3) is the musical fourth; an epogdoic interval (9:8) is the musical whole tone.

> interval of 9/8 [epogdoic], leaving a fraction over, and the interval which this fraction expressed was in the ratio of 256 to 243. And thus the whole mixture out of which he cut these portions was all exhausted by him.

It is obvious how important the principles of harmony and proportion are in Plato's construction of the universe. The three constituents are: "Same" referring to its unchanging, indivisible and eternal nature; "Different" referring to its changing, divisible and physical nature; and "Being" referring to the synthesis of these two seemingly divergent principles. This "World Soul" is thus divided into proportions in accordance to the principles of harmony, and summarized into a "lambda" configuration.[62] This lambda configuration illustrates the distinction between the odd numbers, representing 'Same' and the even numbers, representing 'Different' while Plato's incorporation of the basic principles of the diatonic scale of Philolaus (the fundamental ratios of the fourth, fifth and the octave) and the more convoluted scales of Archytas (which add the harmonic and arithmetic means), synthesizes the fundamental numbers in his harmoniously proportioned "World Soul"[63] :

X **: harmonic mean :: arithmetic mean :** ***Y***

	1	
4/3		**3/2**
3/2		**2**
2		**3**
8/3		**9/2**
3		**6**
4		**9**
16/3		**27/2**
6		**18**
8		**27**

[62] See p. 23 below Chapter 2 of this thesis.

[63] For an detailed explanation of Plato's 'algebraic metaphor' see Robert S. Brumbaugh, *Plato's Mathematical Imagination: The Mathematical Passages in the Dialogues and Their Interpretation* (Bloomington, Indiana and London: Indiana University Press, 1954), pp. 227-229.

Plato now applies these ratios[64] to the construction of the cosmos in an attempt to explain the motion of the sun, moon, planets and stars:

> Next, then, he divided this whole compound along its length into two, and put them together at their centers like an X: then he bent each around to meet itself in a circle, fixing them to themselves and to each other at the point opposite to their first junction, and he made them revolve with a circular motion in the same direction on the same axis, making one of them the outer circle and the other the inner. To the outer movement he gave the name of the nature of the Same, and to the inner that of the Different.[65] ...He gave predominance to the revolution of the Same and Like, for he allowed it to be single and undivided; but he divided the inner revolution six times, making seven unequal circles, each based on the interval of the double and the triple, so that there were three intervals of each kind[66] He commanded the circles to move in directions opposed to one another, three of them at the same speed, the other four traveling at speeds different from those of each other and of the other three, but rationally related to them.[67]

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[64] We can rationalize the ratios in Plato's system, and see them in their musical form, if we multiply all the terms by 6. Then the insertion of harmonic and arithmetic means between terms in the series of doubles, 6, 12, 24, 48, gives us:

6, 8, 9,**12** ,16, 18, **24***,* 32, 36, **48**

and their insertion into the series of triples, 6, 18, 54, 162, gives us:

6, 9, 12, **18**, 27, 36, **54**, 81, 108, **162**

If we combine these numbers into a new series (without doubling numbers) we get:

6, 8, 9, **12**, 16, **18**, **24**, 27, 32, 36, **48**, **54**, 81, 108, **162**

Moving left to right, we can combine each number to its neighbour in the form of a ratio: 6:8, 8:9, 9:12, 12:16, 16:18, *etc.* This series of numbers can now be reinterpreted as a **fourth** (6:8 = 3:4), **tone** (8:9), **fourth** (9:12 = 3:4) – making up one octave**;** **fourth, tone, fourth** – making up a second octave**;** **tone, tone, half-tone (fourth), tone, fourth** – making up a third octave**;** then (oddly) a tone, fifth (2:3)**;** followed by a **fourth, fifth (tone + fourth)** which makes up a fourth octave. In other words, this series of numbers can be translated as three octaves, each divided into two fourths separated by a tone (one of the fourths being subdivided – tone, tone, half-tone) plus a tone and a fifth (which can also be understood as a sixth) and another octave; see Andrew Barker, *Harmonic and Acoustic Theory*, pp. 58-65; Jacques Handschin, "The Timeaus Scale," in: *Musica Disciplina: A Yearbook of the History of Music 4* (1950), pp. 3-42; Gerhard Jahoda, "Die Tonleiter des Timaios - Bild und Abbild," in: *Festschrift Rudolf Haase,* ed. by Werner Schulze (Eisenstadt: Elfriede Rötzer, 1980), pp. 43-80.

[65] The 'Same' corresponds to the outer sphere which are the fixed stars; 'Different' corresponds to the Sun, Moon and planets, each following a particular path of movement**.**

[66] The 'intervals of the double and the triple' correspond to the relative sizes of the seven circles – double intervals: (2:1), (4:2), (8:4) and triple intervals: (3:1), (9:3), (27:9); see Andrew Barker, *Harmonic and Acoustic Theory*, p. 61.

[67] *Timaeus* 36c-d *The Collected Dialogues of Plato,* ed. by Edith Hamilton and Huntington Cairns.

When Plato makes the claim that these speeds are 'rationally related', he is referring not only to the ratios of whole numbers, but also to the fact that these ratios are mathematically and musically apprehensible. As Andrew Barker made abundantly clear above, Plato sees astronomy and harmonics as representing an 'ideal mathematics of motion'; it makes sense, therefore, that Plato would see the seven circles and their movement as forming a harmoniously ordered series. Astronomy and *harmonia* can only, according to Plato, *represent* the Ideal rational order, yet the Pythagorean notion of a 'harmony of the spheres' is clearly alluded to in the above passages.

Although the notion of a 'harmony of spheres' permeated Greek cosmology, and was illustrated with the kind of colourful imagery that we find in Plato's 'Myth of Er'[68], the notion of a 'sounding' harmony of the spheres was contested – in particular, by Aristotle.[69] This Pythagorean theory stems from the association of the "ancient and proverbial seven-stringed lyre'[70] with the notion that the planets are seven in number. As numerical ratios inherent within the harmonic relationship between distance and velocity[71] reveal how such a relationship manifests itself in a concordant musical sound, this same theory, according to the Pythagoreans, can be applied to the

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[68] *Timeaus* 35-36; *The Collected Dialogues of Plato,* ed. by Edith Hamilton and Huntington Cairns. Reference to the 'harmony of the spheres' is also made in Plato's 'Myth of Er' where "Each of the [spindle of Necessity's] circles acted as the vehicle for a Siren. Each Siren, as she stood on one of the circles, sounded a single note, and all eight notes together made a single harmonious sound. Three other women were also sitting on thrones which were evenly spaced around the spindle. They were the Fates, the daughters of Necessity, robed in white, with garlands on their heads; they were Lachesis, Clotho, and Atropos, accompanying the Sirens' song, with Lachesis singing of the past, Clotho of the present, and Atropos of the future"; *Republic* 617b-c; *The Collected Dialogues of Plato,* ed. by Edith Hamilton and Huntington Cairns.

[69] For further reading on Plato's influence on Aristotle's ideology, see Gottfried Marin, "Platons Lehre von der Zahl und ihre Darstellung durch Aristoteles," in: *Zeitschrift für Philosophische Forschung* 7 (1953), pp. 191-203.

[70] Walter Burkert, *Lore and Science,* p. 351; for further reading on the significance of the number 7, see Footnote 29 above in this chapter.

[71] This can be observed from a stringed musical instrument like the lyre.

uniform motion of the planets. Since a musical tone implies a uniform motion, the Pythagoreans assumed that each planet 'sounds' a particular pitch, and that as the numerically relational distances between the planets manifest the same numerical ratios of the diatonic scale, the heavens necessarily resound in a kind of cosmic harmony.[72] In response to this 'sounding' harmony of the spheres, Aristotle writes:

> From all this it is clear that the theory that the movement of the stars produces a harmony, i.e. that the sounds they make are concordant, in spite of the grace and originality with which it has been stated, is nevertheless untrue. Some thinkers suppose that the motion of bodies of that size must produce a noise, since on our earth the motion of bodies far inferior in size and in speed of movement has that effect. Also, when the sun and the moon, they say, and all the stars, so great in number and in size, are moving with so rapid a motion, how should they not produce a sound immensely great? Starting from this argument and from the observation that their speeds, as measured by their distances, are in the same ratios as musical concordances, they assert that the sound given forth by the circular movement of the stars is a harmony. Since, however, it appears unaccountable that we should not hear this music, they explain this by saying that the sound is in our ears from the very moment of birth and is thus indistinguishable from its contrary silence, since sound and silence are discriminated by mutual contrast. What happens to men, then, is just what happens to coppersmiths, who are so accustomed to the noise of the smithy that it makes no difference to them. But, as we said before, melodious and poetical as the theory is, it cannot be a true account of the facts. There is not only the absurdity of our hearing nothing, the ground of which they try to remove, but also the fact that *no effect other than sensitive is produced upon us.* Excessive noises, we know, shatter the solid bodies even of inanimate things...Indeed the reason why we do not hear, and show in our bodies none of the effects of violent force, is easily given: it is that there is no noise.[73]

However, the motive behind Aristotle's response to the Pythagoreans' resounding cosmic harmony is not to undermine their position altogether, but to reinforce his own cosmogony:

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[72] On the Pythagoreans, Aristotle writes: " ...since all other things appeared in their nature to be likenesses of numbers, and numbers to be first in the whole of nature, they came to the belief that the elements of numbers are the elements of all things and that the whole heaven is a harmony and a number; *Metaphysica*, 985b30 - 986a 5, as found in *The Complete Works of Aristotle: the Revised Oxford Translation*, 2 vols., ed. by Jonathan Barnes as part *of Bollingen Series* LXXI/2 (Princeton, New Jersey: Princeton University Press, 1984).

[73] *De caelo,* 290b12 - 291a6, *The Complete Works of Aristotle,* Vol.1, ed. by Jonathan Barnes.

> But not only is the explanation evident; it is also a corroboration of the truth of the views we have advanced. *For the very difficulty which made the Pythagoreans say that the motion of the stars produces a concord corroborates our view.* Bodies which are themselves in motion, produce noise and friction; but those which are attached or fixed to a moving body, as the parts to a ship, can no more create noise, than a ship on a river moving with the stream...Since, therefore, this effect is evidently not produced, it follows that none of them can move with the motion either of animate nature or of constraint.[74]

For Aristotle, philosophy and science are one and the same. As a scientist, he always begins with what is observable. Hence, he makes the claim that "no effect other than *sensitive* [that of the senses] is produced upon us." We can only know what is *sensible*:

> But when that which can hear is actively hearing and that which can sound is sounding, then the actual hearing and the actual sound come about at the same time...Since the actualities of the sensible object and of the sensitive faculty are one actuality in spite of the difference between their modes of being, actual hearing and actual sounding appear and disappear from existence at one and the same moment.[75]

In other words: the *sensible* and the *sensitive* are in themselves potentialities and are actualized in coming together; the tone that results from the actualization of both potentialities coming together is in itself a concord; their relationship, therefore, is one of harmony and proportion – "If voice is a concord, and if the voice and the hearing of it are in one sense one and the same, and if concord is a ratio, hearing as well as what is heard must be a ratio."[76] Although Aristotle's approach does not focus on the quantitative attributes of concordant sound but rather its qualitative nature, he still concedes that inherent within concordant sound is *harmonia*, and that *harmonia* implies the presence of a mean, that is, between the potentiality of that which sounds and the potentiality of the

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[74] *De caelo,*291a7-291a25, *The Complete Works of Aristotle,* Vol.1, ed. by Jonathan Barnes.

[75] *De anima*, 425b29 - 426a19, *The Complete Works of Aristotle,* Vol.1, ed. by Jonathan Barnes.

[76] *De anima.,* 426a28-30, *The Complete Works of Aristotle,* Vol.1, ed. by Jonathan Barnes.

sense-organ which perceives it.[77]

Aristotle's primary goal in his approach to music is to seek out an explanation; he wants to ascertain the 'why' of concords. To do this, he claims that one must begin with what is observed. One must then determine what qualities of a thing are necessary attributes and what are accidental attributes. The former refers to those attributes that are necessarily included in the definition of a thing; the latter refers to those attributes which are not.[78] A thing is then categorized into its particular genus or domain in accordance with its necessary attributes: a triangle will fall under the domain of 'geometry,' a sound will fall under the domain of 'harmonics', and so forth. According to Aristotle, scientific explanation is achieved through a deductive demonstration in the form of a syllogism (a deductive argument whereby a conclusion follows necessarily from two premises). Consequently, one must appeal to the genus (or domain) within which a thing belongs in order to provide a true explanation of that thing – "One cannot, therefore, prove anything by crossing from another genus – e.g. something geometrical by arithmetic."[79] There are, however, exceptions to this general rule, harmonics being one of them:

> Thus with regard to the same science (and with regard to the position of the middle terms) there are these differences between the deduction of the *fact* and that of the *reason why*.
>
> The reason why differs from the fact in another fashion, when each is considered by means of a different science. And such are those which are related to each other in such a way that the one is under the other, e.g. optics to geometry, and mechanics to solid geometry, and *harmonics to arithmetic*, and star-gazing to astronomy. Some of these sciences bear almost the same name – e.g. mathematical

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77 Edward A. Lippman, *Musical Thought in Ancient Greece,* p. 140.

78 For example, a cat is necessarily a mammal, but only accidentally black.

79 *Analytica posteriora,* 75a38-39; *The Complete Works of Aristotle,* Vol. 1, ed. by Jonathan Barnes. For a more thorough argument, see *Analytica posteriora* 75a29 - 75b20, *The Complete Works of Aristotle,* Vol. 1, ed. by Jonathan Barnes.

> and nautical astronomy, and *mathematical and acoustical harmonics*. For here it is for the empirical scientists to know the fact and for the mathematical to know the reason why; for the latter have the demonstrations of the explanations, and often they do not know the fact, just as those who consider the universal often do not know some of the particulars through lack of observation.[80]

Aristotle holds, therefore, that the qualitative attributes of harmonics that are perceived empirically[81] can be described in terms of their quantitative attributes. In other words, one will perceive (hear) the *fact* that an interval is concordant (the octave, for example), and in determining the reason *why* it is concordant, one can ascertain an explanation through mathematics. Such a mathematical explanation can be abstracted and considered independently of its original empirical source. In this way, Aristotle has brought the observation of empirical phenomena (the caused) together with pure mathematics (the cause).

Aristoxenus[82], a student of Aristotle's and considered to be one of the greatest musical authorities of antiquity[83], "speaks as a purer Aristotelian than Aristotle himself."[84] He chose to disregard the purely mathematical approach altogether, treating the science of harmonics as a

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[80] *Ibid.* 78b32 - 79a7.

[81] For further reading on Aristotle's empirical data with regard to acoustics, see Alan Towey, "Aristotle and Alexander On Hearing and Instantaneous Change: A Dilemma in Aristotle's Account of Hearing," in: *The Second Sense: Studies in Hearing and Musical Judgment from Antiquity to the Seventeenth Century*, ed. by Charles Burnett *et al.* as Vol. 22 of *Warburg Institute Surveys and Texts*, ed. by Jill Kraye and W.F. Ryan (London: The Warburg Institute and University of London, 1991), pp. 7-18.

[82] Aristoxenus was the son of a musician, born in Tarentum in the fourth century B.C.E. while Archytas, also from Tarentum, was still alive. He wrote hundreds of books on a wide variety of subjects — most notably music, philosophy , history and a number of biographies on figures such as Pythagoras, Archytas, Socrates and Plato; see Annie Bélis, "Aristoxenus," pp. 1-2; Richard L. Crocker, "Aristoxenus and Greek Mathematics," in: *Aspects of Medieval and Renaissance Music: A Birthday Offering to Gustave Reese*, ed. by Jan La Rue *et al.* (London: Oxford University Press, 1967), pp. 96-110.

8[3] Edward A. Lippman, *Musical Thought in Ancient Greece*, p. 144.

[84] Andrew Barker, *Harmonic and Acoustic Theory,* p. 68.

rigorously empirical discipline[85]: our world can only be understood as phenomena and species of phenomena[86]. Harmonics itself is considered a species of melody (μέλος)[87], as Aristoxenus writes in his *Elementa Harmonica*:

> The branch of study which bears the name of Harmonic is to be regarded as one of the several divisions or special sciences embraced by the general science that concerns itself with Melody. Among these special sciences Harmonic occupies a primary and fundamental position; its subject matter consists of the fundamental principles – all that relates to the theory of scales [*systēmata*: any acceptable series of intervals, even as few as two[88]] and keys [*tonoi*: 'shade' or 'mode']; and this once mastered, our knowledge of the science fulfils every just requirement, because it is in such mastery that its aim consists.[89]

Aristoxenus holds that the study of harmonics is based on the study of empirical phenomena, thereby placing an emphasis on sense perception. We begin with what is perceived and go from there. According to Aristoxenus, there is no exception to Aristotle's general rule of scientific explanation; the empirical phenomena that fall under the title 'harmonics' are not explicable through mathematics. Whereas Aristotle begins with what is perceived and, in the case of harmonics, turns to the science of mathematics to understand the *why*, Aristoxenus stays within the realm of phenomenology: "The fundamental position of the ear remains unassailable; it can neither

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[85] See Malcolm Lichtfield, "Aristoxenos and Emiricism: A Reevaluation Based on His Theories," in: *Journal of Music Theory* 32/1 (Spring 1988), pp. 51-73.

[86] Andrew Barker, *Harmonic and Acoustic Theory.* p. 68.

[87] *Melos* can mean (1) the song which includes melody, rhythm and words, (2) the melody itself or (3) the melodic series or scale on which a melody is based; see Andrew Barker *Harmonic and Acoustic Theory*, p. 126.

[88] *Ibid.* p. 126.

[89] Aristoxenus, *Harmonica*, Book 1; Henry Stewart Macran, ed., ΑΡΙΣΤΟΞΕΝΟΥ ΑΡΜΟΝΙΚΑ ΣΤΟΙΧΕΙΑ*: The Harmonics of Aristoxenus* (Oxford: Clarendon Press, 1902), p. 165.

be invaded by reason nor reduced to mathematical or physical principles."[90] He is not concerned with the mathematical ratios inherent in the physical 'making' of a sound – the *why*; he is only interested in the object of perception itself – the *fact*.

Aristoxenus chooses melody as his object of study. Best represented in the human voice, melody, like speech, it a continuum which is made of smaller constituents and is bound by the principles of harmonics. He writes:

> Continuity in melody seems in its nature to correspond to that continuity in speech which is observable in the collocation of the letters. In speaking, the voice by a natural law places one letter first in each syllable, another second, another third, another fourth, and so on. This is done in no random order; rather, the growth of the whole from the parts follows a natural law. Similarly in singing, the voice seems to arrange its intervals and notes on a principle of continuity, observing a natural law of collocation, and not placing any interval at random after any other, whether equal or unequal.[91]

Unlike Plato, who considered music produced by musicians to be a mere distorted shadow of the eternal Form of Music, Aristoxenus would have sense perception woven into the very fabric of music: sense perception comes first, and is therefore vital to the student of musical science. If one begins with faulty data, then it is impossible to pursue those questions that transcend what is perceived.[92] A scientific approach to music, according to Aristoxenus, involves two distinct and autonomous faculties, that of hearing and that of the intellect. Each faculty performs a vital role – "by the former we judge the magnitudes of the intervals, by the latter we contemplate the functions

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[90] Edward A. Lippman, *Musical Thought in Ancient Greece,* p. 150.

[91] Aristoxenus , *Harmonic Elements,* as cited in Edward A. Lippman, *Musical Thought in Ancient Greece*, p. 148.

[92] Edward A. Lippman, *Musical Thought in Ancient Greece*, p. 149.

of the notes."[93] It is the job of one who undertakes the science of harmonics to observe the object of perception (melody, or μέλος), clarify the concepts and terminology that specifically fall under the domain of 'harmonics', and classify and categorize the phenomena perceived.

It has been established that sense perception takes a primary role in Aristoxenus' methodology. Consequently, what determines 'concord' and 'discord' in melody cannot be found merely in numerical ratios. The size of the intervals and the succession of pitches is not determined directly by mathematical principles, but instead by the consistent harmonic laws that govern the intervals and pitches to create what the ear *hears* as concordance. Yet, what governs these laws? It is here that his approach seems incomplete.

Aristoxenus holds that concord and discord are simply relational ideas – there is no reason to delve into the *why* (as Aristotle did) of musical concordance, the ear will perceive it as such, or not. For example, if one takes a purely mathematical approach and merely applies the Pythagorean mathematical principles to the diatonic scale, then inconsistencies occur.[94] Aristoxenus claims that what the science of harmonics boils down to is what one *hears.* Intervals are consonant, not because of governing mathematical principles (that may prove to be inconsistent), but because we hear them as consonant. Dissonance is understood in *relation* to consonance – the less consonant the interval is to the ear, the more dissonant the interval is perceived to be, and the more displeasing it is for the

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[93] *Ibid.* p. 149.

[94] The measure of the half-tone inevitably changes as one climbs by fifths (the *diapente*). After one has completed seven octaves and has returned to the original note name, there is a discrepancy in pitch. This discrepancy is known as the Pythagorean comma. The only way that one can return to the original note *acoustically* is to have all of the half-tones of equal measure, hence the equal temperament that we are accustomed to now. Such inconsistencies in Pythagorean music theory do not, however, preclude that there are no governing mathematical principles in the science of harmonics; see Chapter 3 (Footnote 67) below in this thesis. For a complete explication of the Greek musical scale, see Andrew Barker, *Harmonic and Acoustic Theory*, pp. 46-52; see also André Barbera, "Pythagorean Scale," in: *The New Harvard Dictionary of Music*, ed. by Don Michael Randel (Cambridge: Harvard University Press, 1986), p. 673.

listener. For Aristoxenus, it makes far more sense to determine the degree of consonance in a relational way from what we have perceived, rather than through a recognition of mathematical consistencies or inconsistencies. Whether or not the numerical ratios work, the ear will always hear what it hears.

Aristoxenus kept the science of harmonics exclusively within the empirical realm and he was extremely successful in creating a methodological theory of music for practicing musicians. He was extremely prolific, and, starting from the very basics, was the first to make an attempt at systemizing music in all of its complexities, articulating such nuances as *tonoi* (referring to the different shades of *genera,* or modes, of music) and melodic *systēmata*. Very few revisions were made to his compendium by later harmonic theorists, and our understanding of Greek music theory today owes much to his principal work, Αρμονικα Στοιχεια,, or *Harmonic Elements*.[95] However, if Aristoxenus were to investigate the laws of harmony further – if he were to inquire more into the *why* of concordance and not merely the *fact* of it – then perhaps he would not have been able to deny the correlation of concordance with the consistency of specific numerical ratios. In other words, an interval deemed 'consonant' by a musician because of its sweetness, is consonant – and sweet – *because of* governing mathematical principles.

As this chapter has revealed, the tension that existed between the purely mathematical approach and the strictly empirical approach to the science of harmonics in Greek Antiquity was never resolved.[96] Yet, despite fact that the purely mathematical approach of the Pythagoreans and

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[95] See "Aristoxenus Redeemed in the Renaissance," in: Claude V. Palisca*, Studies in the History of Italian Music and Music Theory* (Oxford: Clarendon Press, 1994), pp. 189-199; Annie Bélis, "Aristoxenus," pp.1-2; Andrew Barker, *Harmonic and Acoustic Theory,* p. 124.

[96] The closest any theorist came to resolving the tension between the two is Ptolemy, who will be discussed in Chapter 3 of this thesis.

purely empirical approach of Aristoxenus represent two distinct and core components to this tension, it was generally understood that the discipline of music was as rigorous as any science. It was not until the Renaissance that controversy arose regarding music's place as one of the four branches of mathematics found in the *quadrivium* (along with geometry, arithmetic and astronomy). Boethius' *De institutione musica* (*Fundamentals of Music*), a mathematically based treatise of music theory which served as *the* authoritative text in musical study throughout the Middle Ages, underwent a revival in the Renaissance which kindled two opposite reactions: one which praised his purely theoretical approach, another which sparked an anti-theoretical movement.[97] The controversy that ensued resulted in a complete about-face with regard to the discipline of music. No longer was music viewed as a rigorous science, with underpinnings of mathematical theory. By the end of the sixteenth century, music came to be seen in a strictly aesthetic light; its epicenter shifting from *musica theorica* to *musica practica*, from the theory - based musician to the practicing one.[98] As the following chapters will reveal, however, this shift of focus from the theoretical to the practical does not preclude the absence of inherent and governing mathematical principles.

[97] See Claude V. Palisca, "Theory, theorists," p. 371; Claude V. Palicsa, "Boethius in the Renaissance," pp. 259-280.

[98] Note that as early as the late fifteenth century, musical scholars such as Franchino Gaffurio devoted separate volumes to *musica theorica*, see his *Theorica musice* (1492) and *Practica musicae* (1496); see also Claude V. Palisca, "Theory, theorists," pp. 371-373; Clement A. Miller, trans. *Franchinus Gaffurius*.

Chapter 3
An Extension of Pythagorean Thought into the Middle Ages and the Ensuing Controversy: From Boethius to Descartes

The distinction between *musica theorica* and *musica practica* surfaces in late antiquity and becomes the focus of much debate in the Renaissance.[1] The ‘bible’ of *musica theorica,* and the source of inspiration and contention for many music scholars of that time is Boethius’ *De institutione musica* (*Fundamentals of Music*)[2]. Written at the turn of the sixth century C.E., this text provides modern-day scholars with a great insight into Western Medieval music theory and how it evolved[3], as it was acknowledged as the primary authoritative source in music theory (especially with regard to music’s mathematical nature) during the Middle Ages. Furthermore, the *De institutione musica* is a work based on the premise of reconciliation between faith and reason; it is no wonder that this seminal treatise played a central role in the discipline of music between the ninth and sixteenth centuries, when philosophy’s ultimate purpose was seen as a means of accomplishing beatification through the faculties of the intellect. It is of the utmost importance when approaching this particular work that one takes into consideration the context from which it was written, for Boethius’ treatise on music is an attempt to *reveal* the basic truths of mathematics and music.[4] One of his earliest works, the *De insitutione musica* “was intended to be read along

[1] See Claude V. Palisca, “Theory, theorists,” p. 371; Claude V. Palisca, “Boethius in the Renaissance,” ed. by André Barbera, pp. 259-280.

[2] Two authoritative texts on Boethius’ *De institutione musica* are: Roger L. Bragard. “Les sources du ‘De institutione musica’ de Boèce,” (Unpublished Ph.D. Dissertation, Université de Liège, 1926); Calvin Bower, “Boethius’ De institutione musica: A Handlist of Manuscripts,” in: *Scriptorium* 42 (1988), pp. 205-251.

[3] Claude V. Palisca, “Preface,” in: Boethius, *Fundamentals of Music*, trans. by Calvin Bower, p. xiv.

[4] Calvin Bower, trans., Boethius, *Fundamentals of Music,* p. xxiv.

with the *De institutione arithmetica*[5] and may have been one of four works setting out the foundations of Platonic scientific education: arithmetic, music, geometry, and astronomy [the *quadrivium*]."[6] What will follow in this chapter is an explication of the shift that occurred in the discipline of music during the Renaissance, from the *musica theorica* to the *musica practica*, with Boethius' monumental *De institutione musica* serving as the pivotal point.

Boethius was born into a wealthy, prominent family of Rome around 475 C.E. and received a well-rounded and liberal education. Although most famous for his *Consolation of Philosophy*[7], he is also known (and appreciated) for undertaking the daunting task of translating from Greek into Latin (with commentary) the works of Euclid, the Pythagoreans, Nicomachus, Ptolemy, Plato and Aristotle[8]. As Calvin Bower notes:

> Boethius carried the genre [of translating Greek scholars of antiquity into Latin[9]] to new levels of rigor and thoroughness. Written for a cultural elite already initiated

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[5] Books 2 and 3 of *Fundamentals of Music* make constant reference to *Fundamentals of Arithmetic*, as logical demonstrations of the mathematical (and abstract) core of music theory is presented. For a comprehensive discussion on Boethius' two seminal treatises, see Gottfried Friedlein, ed., *Anicii Manlii Torquati Severini Boetii: Institutione arithmetica libri duo e Institutione musica libri quinque accedit Geometria quae fertur Boetii* (Frankfurt am Main: Minerva, 1966 is reprint of Leipzig: B.G. Teubner, 1867); John Caldwell, "The *De institutione arithmetica* and the *De institutione musica*," in: *Boethius: His Life, Thought and Influence,* ed. by Margaret Gibson (Oxford: Basil Blackwell, 1981), pp. 135-154; Michael Masi, "The Influence of Boethius' De Arithmetica: On Late Medieval Mathematics," in: *Boethius and the Liberal Arts,* ed. by Michael Masi as, Vol. 18 of *Utah Studies in Literature and Linguistics* (Berne and Frankfurt am Main: Peter Lang, 1981), pp. 81-95.

[6] Calvin Bower, trans., Boethius, *Fundamentals of Music*, p. xix.

[7] This work was written while in prison awaiting execution (524) and is concerned, among other things, with the problem of evil. For a comprehensive study of the *Consolation of Philosophy,* see Henry Chadwick, *Boethius: The Consolations of Music, Logic, Theology and Philosophy* (Oxford: Clarendon Press, 1983 is a reprint of 1981); David S. Chamberlain, "Philosophy of Music in the 'Consolatio' of Boethius," in: *Speculum* 45 (1970), pp. 80-97.

[8] His vast collection of translations include: Aristotle's *Categories*; Ptolemy's *Haromoncia*; Nicomachus' *Eisagoge arithmetica and Eisagoge musica;* Plato's *Symposium and Timaeus*; Euclid's *Elementa*; among others, see Boethius, *Fundamentals of Music*, trans. by Calvin Bower, pp. xxiv-xxix.

[9] Claude V. Palisca quotes fifteenth-century scholar Johannes Gallicus as saying "that Music, which the so often mentioned Boethius turned into Latin from the Greek" (ea namque musica, quam totiens allegatus Boethius de Graeco vertit in latinum); see Claude V. Palisca, "Boethius in the Renaissance," p. 260.

> into philosophical literature, Boethius' mathematical and logical works represent one of the most notable projects in intellectual history of preserving and transmitting a corpus of knowledge from one culture to another.[10]

It was not until the ninth century that liberal learning resurfaced in the West. As Boethius was one of the few reliable sources of Greek texts translated into Latin, he inadvertently became the pivotal source of Greek antiquity offered in Europe for many centuries thereafter.[11]

The *Fundamentals of Music* is not merely a translation of Greek texts, most notably Nicomachus[12] and Ptolemy [13]of the second century C.E., but an attempt to incite his reader, through music theory, to pursue the study of philosophy. For Boethius the true musician is not one who sings, or masters his instrument, or even composes; the true musician is the philosopher who "masters and applies the speculative principles of the discipline."[14] This is the underlying thesis that guides Boethius through each of the five books of this treatise. To fully understand the controversy that surfaced during the Renaissance, a brief synopsis of these five books is in order.

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[10] Calvin Bower, trans., Boethius, *Fundamentals of Music,* p. xx.

[11] Consequently, his treatises were not contested until the fifteenth century, when scholars took a renewed interest in reading sources in their original Greek; see Claude V. Palisca, "Theory, theorists*,"* p. 363.

[12] Primarily Nicomachus' *Eisagoge arithmetica and Eisagoge musica*. For an English translation of the *Eisagoge arithmetica*, see Frank Egelston Robbins and Louis Charles Karpinski, *Introduction to Arithmetic, with Studies in Greek Arithmetic*, translated by Martin Luther D'Ooge, Vol 16 of *University of Michigan Humanistic Studies Series* (New York: Johnson Reprint, 1972 is reprint of 1926); see also, George Johnson, *The Arithmetic Philosophy of Nicomachus of Gerasa* (Lancaster, Pennsylvania: Press of the New Era Printing Company, 1916). For further readings on Boethius and Nicomachus, see Calvin Bower, "Boethius and Nicomachus: An Essay Concerning the Sources of *De institutione musica,*" in: *Vivarium: An International Journal for the Philosophy and Intellectual Life of the Middle Ages and Renaissance* 16 (1978), pp.1-45; Flora Rose Levin, *The Harmonics of Nicomachus and the Pythagorean Tradition*, Vol.1 of *American Classical Studies* (University Park, Pennsylvania: The American Philological Association, 1975); Jay Kappraff, "The *Arithmetic* of Nicomachus of Gerasaand and its Applications to Systems of Proportion," *Nexus Network Journal*, 2/4 (October, 2002), URL = http://www.nexusjournal.com/kappraff.html; Thomas J. Mathiesen, *Apollo's Lyre,* pp. 390-411.

[13] Primarily Ptolemy's *Harmonics.* For a good overview of Ptolemy, see Thomas J. Mathiesen, *Appollo's Lyre*, pp. 429-495.

[14] Calvin Bower, trans., Boethius, *Fundamentals of Music,* p. xxxi.

The prologue of Book I sets the climate of the treatise. Combining myth[15] and fact,[16] Boethius draws the reader into the Pythagorean mind-set[17]: that because of its mathematical nature, music not only permeates our own human existence, but is a fundamental – and governing – constituent of the universe itself. The purpose of the first book is to introduce the basic tenets of Pythagorean music theory[18] (intervals and ratios based on Pythagorean principles) which will provide the basic building blocks of the books to come. In an extremely important section of this book[19] Boethius introduces the theory of consonances:

> When two strings, one of which is lower, are stretched and struck at the same time, and they produce, so to speak, an intermingled and sweet sound, and the two pitches coalesce into one as if linked together, then that which is called 'consonance' occurs.[20] When, on the other hand, they are struck at the same time and each desires to go its own way, and they do not bring together a sweet sound in the ear, a single sound composed of two, then this is what is called 'dissonance'."[21]

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15 In the prologue of Book I Boethius relays the story of Pythagoras' calming a drunken youth who was apparently incited by the Phrygian mode into acts of violence. Pythagoras placated him into a serene state of mind by playing a mode known for its calming effect.

16 For example, Boethius briefly discusses the use of various modes that are used to manipulate emotion, and how they have been utilized in the education of children – Plato discusses this in his *Republic* 398c - 403c; see *The Collected Dialogues of Plato,* ed. by Edith Hamilton and Huntington Cairns.

17 For further reading see Jean Edmiston, "Boethius on Pythagorean Music," in: *The Music Review* 35 (1974), pp. 179-184.

18 For a more in depth discussion on Pythagorean music, see Barbara Münxelhaus, *Pythagoras musicus.*

19 Boethius, *Fundamentals of Music*, trans. by Calvin Bower, Chapters 28-32 of Book I [220-223].

20 The 'intermingling' of the high and low which produces a sweet sound is the coalescing of two sound waves in a concordant mathematical relationship; see Footnote 61 below. Boethius would not have known this, however, as experiments in sound-waves and frequencies did not become commonplace and documented until the Renaissance. For a comprehensive discussion of sound perception in the Middle Ages, see Charles Burnett, "Sound and Its Perception in the Middle Ages," in : *The Second Sense: Studies in Hearing and Musical Judgment from Antiquity to the Seventeenth Century*, ed. by Jill Kraye and W.F. Ryan (London: The Warburg Institute and University of London, 1991), pp. 43-70.

21 Boethius, *Fundamentals of Music*, trans. by Calvin Bower, Book 1, Chapter 28 [220]; see Footnote 61 below in Chapter 3 of this thesis.

He goes on to discuss the mathematical ratios inherent in the 'sweet sound' of consonance, and how these ratios are commensurable with each other and how some consonances take precedence over others, depending upon how easily the intervals' inherent ratios are grasped by the intellect (hence, the octave [2:1] takes precedence over, say, the fifth [3:2]).[22] He ends this first book with a bold claim that beautifully illustrates his position: " How much nobler, then, is the study of music as a rational discipline than is composition and performance! It is as much nobler as the mind is superior to the body; for devoid of reason, one remains in servitude."[23]

Books II and III are "relentlessly technical,"[24] making continuous reference to *Fundamentals of Arithmetic*, firmly placing music as one of the four mathematical disciplines of the *quadrivium*, and demonstrating the quantitative basis for each interval introduced in Book I. At the heart of Book IV lies the division of the monochord,[25] based again on Pythagorean mathematical principles and the Pythagorean division of the monochord, and Boethius' claim that reason does not contradict sense perception (that is, the correlation of consonance and dissonance with mathematical ratio). Boethius' description of Greek notation and the division of the monochord help to illustrate the

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[22] It is important to note here that the octave, and the fifth, are found to play a vital role in all music cross-culturally. See Chapter 4 of this thesis.

[23] Boethius, *Fundamentals of Music*, trans. by Calvin Bower , Book 1, Chapter 34 [224-227].

[24] Calvin Bower, trans., Boethius, *Fundamentals of Music,* p. xxxi.

[25] The monochord can be described as a zither with a single string stretched over a rectangular sound box, with a movable bridge. The divisions of the string according to mathematical ratios (corresponding to various intervals) are marked on the sound box. For further readings on the division of the monochord, see Sigfrid Wantzloeben, *Das Monochord als Instrument und als System entwicklungsgeschichtlich dargestellt* (Halle an der Saale: Max Niemeyer, 1911); Thomas J. Mathiesen, "An Annotated Translation of Euclid's Division of a Monochord," in: *Journal of Music Theory* 19 (1975), pp. 236-258; Joseph F. Smith, "The Medieval Monochord," in: *Journal of Musicological Research* 5 (1984), pp. 1-33.

system of *tonoi* (*genera*, or shades)[26] or what we would understand as (and he calls) 'modes'.

Book V is for the most part an exposition of Ptolemy's *Harmonica*[27]. Boethius wants to prove that rational thought and the senses are *not* irreconcilable[28] (hence, *musica theorica* is vital to our understanding of what the ear *hears* as consonant) and Boethius will use Ptolemy's *Harmonica* to back up his claim. Boethius' recurring argument – that judgements of reason are compatible with those determined by the senses – crystallizes with Ptolemy's harmonic theory. As Ptolemy relies on both reason and the senses[29], so does Boethius make the following claim:

> Harmonics is the faculty that weighs differences between high and low sounds using the sense of hearing and reason. For sense and reason are, as it were, particular instruments [or criteria[30]] for the faculty of harmonics...[the sense of hearing] grasps differences between neighboring pitches by means of the sense, [reason] considers the integral measure and quantity of these same differences.[31]

Up until now, the *Fundamentals of Music* has focused solely on Pythagorean doctrine, and the only cited authoritative critic of the Pythagoreans thus far has been Ptolemy. It is in Book V,

[26] See Archytas and Aristoxenus above in Chapter 2 of this thesis. For a full explication of *tonoi*, or τόνος, see Solon Michaelidess, *The Music of Ancient Greece,* pp. 335-340; Jon Solomon, "Towards a History of Tonoi" (part of Round Table: "The Ancient Greek Harmoniai, Tonoi and Octave Species in Theory and Practice," organized and chaired by Claude V. Palisca at the National Meeting of the American Musicological Society in Louisville, Kentucky, Oct. 27, 1983), in: *The Journal of Musicology: A Quarterly Review of Music History, Criticism, Analysis and Performance Practice* 3 (1984), pp. 242-251.

[27] Ptolemy worked in Egypt in the second century C.E. For a good overview of Ptolemy, see Andrew Barker, *Harmonic and Acoustic Theory*, p. 270.

[28] In proving the reconciliation of the senses with the faculty of reason in musical experience, Boethius supports his own premise of the reconciliation of faith and reason mentioned above in Chapter 3 of this thesis.

[29] However, Ptolemy leans toward reason; see Ptolemy *Harmonics,* as cited in Calvin Bower, trans., Boethius, *Fundamentals of Music,* p. xxxv.

[30] In the first four books of Boethius, *Fundamentals of Music,* music treated as one of the four mathematical disciplines necessary in the study of philosophy; in this passage, Boethius borrows from Ptolomy's *Harmonica* 1.1 as he posits 'hearing' *and* 'reason' as necessary criteria for the study of Harmonics; see Calvin Bower, trans., Boethius, *Fundamentals of Music*, p.163 (Footnote 3).

[31] Boethius, *Fundamentals of Music,* trans. by Calvin Bower, Book 5, Chapter 2 [352-355].

however, that Boethius chooses to discuss "those matters about which ancient musical scholars express contradictory opinions."[32] Once again, the tension between the purely mathematical approach and the strictly empirical approach resurfaces. These 'contradictory opinions' can be divided into two camps: the Pythagorean (strictly mathematical), and the Aristoxenian (strictly empirical). Aristoxenus maintains that reason plays only a secondary role to the senses; it is the senses that we rely on to determine the measure of intervals and what is or is not consonant. The Pythagoreans place the senses secondary to reason; although they concede that one may use empirical data as a tool to judge truths[33], they claim that such data is fallible and can be misleading. Hence an interval that is seemingly sweet is rejected as dissonant if its mathematical ratio does not comply with those ratios the Pythagoreans have classified as 'consonant'.[34]

Ptolemy came the closest to resolving the ongoing tension between mathematical ratios and the musician's ear. However, if one were to place Ptolemy in one camp or the other, he would definitely be considered a Pythagorean. After all, Boethius devotes an entire chapter to Ptolemy in order to justify his own claim that, while the senses play a fundamental role in music, reason is music's governing principle. However, as Boethius puts it: "Ptolemy is quite critical of Aristoxenus and the Pythagoreans ... for Aristoxenus does not trust reason at all but only the senses, while the Pythagoreans are too little concerned with the senses and too much concerned with the ratios yielded

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[32] *Ibid.,* Book 5, Chapter 1 [351].

[33] See Walter Burkert, *Lore and Science,* pp. 369-386; Andrew Barker, *Harmonic and Acoustic Theory*, pp. 39-42; Barbara Münxelhaus, *Pythagoras musicus*.

[34] An example of this is the Pythagorean rejection of the consonant sounding interval of the diapason-plus-diatessaron (the octave plus the fourth) based on its mathematical ratio. Ptolemy felt that the Pythagoreans' mathematical classifications were too limited.

by reason."[35] According to Boethius, Ptolemy takes a far more broad-minded position espousing the virtues of both the senses and reason (with emphasis on reason), making the claim that intervals sounding consonant do in fact have mathematical ratios that are consistent. He agrees with the Pythagoreans that the difference between high and low pitches is quantitative and that intervals can be expressed mathematically, but his classification of intervals goes far beyond anything they would have considered acceptable.[36] Ptolemy translates the ratios found in the Pythagorean system into the lowest numbered ratios that correspond to what the musician's ear hears as consonant. Consequently, his categories include ratios that the Pythagoreans would have rejected – but do sound pleasing to the ear.[37] For instance, the Pythagorean translation of the interval of a 'major third' into a mathematical ratio (deciphered mathematically as the product of two tones[38]) is 81:64. Yet the musician would tune a consonant sounding third to the simpler ratio of 5:4 (or 80:64).[39] Ptolemy did not reject this simpler ratio as imperfect because it was not arrived at in a purely mathematical way; the ratio 5:4 sounds sweet to the ear. As far as Ptolemy was concerned, the more simplex the ratio, the more consonant the interval. Further, the mathematical ratios describing the

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[35] Boethius, *Fundamentals of Music,* trans. by Calvin Bower, Book 5, Chapter 3 {355]. In Chapters 13-18, Boethius discusses Ptolemy's criticism of both Aristoxenus and Archytas with regard to their division of the diapason (octave) and the diatessaron (fourth).

[36] *Ibid.,* Book Five, Chapters 11 and 12. Boethius discusses what he considers Ptolemy's reply to the Pythagorean evaluation and classification of consonances to support his own position with regard to the mathematical rationale implicit within intervals. However, it should be noted that Boethius' conflation of two separate chapters in Ptolemy's *Harmonica* somewhat compromises Ptolemy's position; see Calvin Bower, trans., Boethius, *Fundamentals of Music*, p. 170 (Footnote 31).

[37] This is reminiscent of Archytas; see Chapter 2 above in this thesis.

[38] See Chapter 2 above in this thesis.

[39] Note also Archytas in Chapter 2 abovein this thesis – Ptolemy merely accepted the ratio, while Archytas sought to justify it mathematically through arithmetic and harmonic means; see Andrew Barker, *Harmonic and Acoustic Theory*, pp. 46-52.

musician's tuning were as acceptable, perhaps even *more* acceptable, than the ratios arrived at purely mathematically. Consequently, he has a much broader classification of musical intervals that can be translated into consistent numerical ratios.[40]

Ptolemy successfully concedes the importance of physical perception, without jeopardizing the claim that we arrive at an *understanding* of the laws that govern harmonic structure through mathematical reasoning. In other words, he did not neglect the *why* as Aristoxenus did. This lends support to Boethius' own position. Boethius understands the true musician as one who dwells on the mathematical principles inherent within music, for it is mathematical reasoning that allows one to make sense of the sweet sounds that are perceived by the ear. Hence a reconciliation between our ability to speculate (reason) and what is experienced (faith) has been accomplished. Speculation, however, is that which brings one closer to truth. This is why the main objective in his treatise of music theory is to pursue philosophy, rather than becoming an accomplished musician.[41]

Boethius' work was the primary source of music theory for students during the Middle Ages and was available in every monastic library.[42] Although Boethius faded into the shadows somewhat

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[40] See Boethius, *Fundamentals of Music,* trans. by Calvin Bower, Book Five, Chapter 11.

[41] Guido of Arezzo, an eleventh-century music theorist, was obviously familiar with Boethius (he recounts the Pythagorean numerical ratios of the consonants). Arezzo makes the claim, however, that Boethius' treatise of music was useful for philosophers, but not for singers. He wrote his *Micrologus* around 1026 as a manual to train choirs, and is most noted for implementing the 'Guidonian solmization' (*ut, re, mi, fa, sol la*) which helps singers recognize where the tones and semitones appear in the scales they are singing. His work was completely original for its time and fundamentally practical; see Claude V. Palisca, "Theory, theorists," pp. 365-356.

[42] Although much was written during the Middle Ages with regard to music theory, and change and progression in music for voice and instruments is apparent, it was, for the most part, over-shadowed by *De institutione musica*, and the underlying system of scales and modes set out by Boethius was the methodological model used by Western scholars well into the twelfth century. For an overview of the progression of plainchant, and the emergence of polyphony (and so forth) from Boethius through to the fifteenth century (accompanied by an impressive bibliography), see Claude V. Palisca, "Theory, theorists," pp. 362-370.

during the thirteenth and fourteenth centuries as contemporary scholars came to the forefront[43], with the Renaissance his work was once again in the spotlight and scholars were newly incited to search for the texts and authors mentioned in his treatise.[44] Two opposite reactions ensued: an attempt to keep the consonances 'pure' in accordance with Pythagorean and Boethian ratios (*musica theorica)* and an anti-theoretical movement that aspired towards a broader scope of 'consonance' that relied solely on the musician's ear (*musica practica*).

Up until the Renaissance, theoretical music occupied a prestigious place among the four mathematical disciplines of the *quadrivium*, and remained separate and distinct from the world of the practicing musician.[45] With the rebirth of Greek humanism, the early stages of the Renaissance

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[43] See, for example, Johannes de Garlandia, *De mensurabili musica* (*ca.* 1225); Franco of Cologne's *Ars cantus mensurabilis* (*ca.* 1250); the treatise of Anonymus 4 (*ca.* 1270-1280); the treatise of the St.Emmeram Anonymus (1279); Hieronymus de Moravia's *Tractatus de musica* (1272); Walter Odington's *Summa de speculatione musice* (early fourteenth century); and Jacobus of Liège's *Speculum musice* (*ca.* 1325). For Latin editions of the respective treatises, see Erich Reimer, ed. *Johannes de Garlandia: De mensurabili musica – Kritische Edition mit Kommentar und Interpretation der Notationslehre*, 2 parts, Vol. 11 of *Beihefte zum Archiv für Musikwissenschaft*, ed. by Hans Heinrich Eggebrecht *et al.* (Wiesbaden: Franz Steiner, 1972); Gilbert Reaney and Andre Gilles, eds., *Franconis de Colonia: Ars cantus mensurabilis*, Vol. 18 of *Corpus Scriptorum de Musica*, ed. by Armen Carapetyan (American Institute of Amusicology, 1974); Fritz Reckow, ed., *Der Musiktraktat des Anonymous 4*, 2 parts, Vol. 4 of *Beihefte zum Archiv für Musikwissenschaft*, ed. by Hans Heinrich Eggebrecht *et al.* (Wiesbaden: Franz Steiner, 1967); Jeremy Yudkin, *De musica mensurata: The Anonymous of St. Emmeram – Complete Critical Edition, Translation and Commentary*, part of *Music: Scholarship and Performance*, ed. by Thomas Binkley (Bloomington and Indianapolis, Indiana: Indiana University Press, 1990); S. M. Cserba, ed., *Hieronymus de Moravia O.P.: Tractatus de musica*, Vol. 2 of *Freiburger Studien zur Musikwissenschaft* (Regensburg, 1935); Frederick F. Hammond, ed., *Walter Odington: Summa de speculatione musicae,* Vol. 14 of *Corpus Scriptorum de Musica*, ed. by Armen Carapetyan (American Institute of Musicology, 1970); Roger Bragard, ed., *Jacobi Leodiensis: Speculum musicae*, Vol. 3 of *Corpus Scriptorum de Musica*, ed. by Armen Carapetyan (American Institute of Musicology, 1961). For a good overview of thirteenth and fourteenth century musical scholarship, see Claude V. Palisca, "Theory, theorists,"; Palisca further notes in this article that: "Reading the treatises of the thirteenth and fourteenth centuries leads one to question how much of Boethius was studied or understood during those years of swiftly changing musical practices or how relevant the book was considered"; see, *ibid.* p. 371.

[44] Claude V. Palisca, *Girolamo Mei: Letters on Ancient and Modern Music,* p.40; Claude V. Palisca, "Boethius in the Renaissance," p. 259.

[45] Claude V. Palisca, "Scientific Empiricism," p. 92.

witnessed a brief synthesis of *musica theorica* and *musica practica.*[46] However, toward the end of the sixteenth century "...the *quadrivium* exploded from its inner stresses and expanding constituents and the two disciplines, musical art and musical science, began to acquire their separate modern identities."[47] The scholars whose ongoing polemics best illustrate this contentious climate are Zarlino[48] and Galilei[49], and Mersenne[50] and Descartes[51].

Gioseffo Zarlino firmly held that music is a manifestation of numbers, and that it is only

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[46] *Ibid*., p. 92. This was aspired to by such music scholars and humanist patrons as Johannes Tinctoris, Franchinus Gaffurius, Girolamo Mei and Count Giovanni Bardi (who established the Florentine 'Camerata", an informal gathering of noblemen and musicians who, in the Greek humanistic fashion, would discuss poetry, music and philosophy); see Claude V. Palisca, *Girolamo Mei: Letters on Ancient and Modern Music*, Chapters I to IV.

[47] *Ibid.* p. 93.

[48] Sixteenth-century music theorist and composer, Gioseffo Zarlino was noted for his tenacious adherence to the Pythagorean tuning and classification of consonances, although in his treatise, *Le istitutioni harmonich* (1558), he advocates the syntonic diatonic tuning of Ptolemy; see Gioseffo Zarlino, *The Art of Counterpoint*, trans. by Guy A. Marco and Claude V. Palisca as part of *Music Theory Translation Series*, ed. by Claude V. Palisca (New Haven, Connecticut and London: Yale University Press, 1976 is second printing of 1968), Part III of *Le Istitutioni harmoniche* (1558); Claude V. Palisca, "Theory, theorists," pp. 373-374.

[49] Vincenzo Galilei, a sixteenth-century Florentine and father of astronomer Galileo Galilei, was a theorist, lutenist, and student of Zarlino (and later became Zarlino's severest critic); see Claude V. Palisca, "Scientific Empiricism," p. 120. For an in depth and comprehensive discussion of Galilei and his works, see Claude V. Palisca, trans., *Vincenzo Galilei: Dialogue on Ancient and Modern Music,* trans. with introduction and notes by Claude V. Palisca as part of the *Music Theory Translation Series*, ed. by Claude V. Palisca (New Haven, Connecticut and London: Yale University Press, 2003). For an interesting explication of Galilei and Kepler on Cosmology, see Owen Gingerich, "Kepler, Galilei, and the Harmony of the World," in: *Music and Science in the Age of Galileo*, ed. by Victor Coelho as Vol. 51 of *The University of Western Ontario Series in Philosophy of Science*, ed. by Robert E. Butts (Dordrecht, Netherlands; Boston, Massachusetts; and London: Kuwer Academic Publishers, 1992), pp. 45-63.

[50] Marin Mersenne (1588-1648) was a mathematician, philosopher and scientist. He corresponded not only with Descartes, but with all the leading scientists of his day, including Pascal, Fermat and Hobbes. His correspondences served as a kind of network of information within the academic community in Europe. In his writings he approaches music as a scientific discipline, experimenting and measuring the physical properties of sound. His most complete treatise on music was *Harmonie universelle* (Paris, France: S. Cramoisy, 1636-1637). For two examples of correspondences between Mersenne and Descartes, see Claude V. Palisca, "Scientific Empiricism," p. 98 (Footnotes 17 and 18).

[51] René Descartes (1596-1650) was the "chief architect of the seventeenth century intellectual revolution which destabilized the traditional doctrines of Medieval and Renaissance scholasticism..."; see John Cottingham, *The Oxford Companion to Philosophy*, ed. by Ted Honderich (Oxford: Oxford University Press, 1995), p. 188. His influence was strongly felt in the moving trend of music toward subjectivism and aesthetics, as he firmly held that consonances and tuning systems were reliant on the subjective ear of the musician.

through studying these 'sounding' numbers that one may understand the tones and ratios that constitute what we understand to be music. In his *Institutioni harmoniche* (1558) he proposed that the *numero senario* (literally 'number six') constitutes all possible consonances within the musical spectrum, and that the division of strings into simple ratios attests to the nature of harmony, and produces all intervals that composers call 'perfect'.[52] In monophony, such intervals as the major and minor thirds and sixths seldom arose, if at all; with the development of polyphony (through the Middle Ages and the Renaissance) there arose a need to justify these intervals as consonances. It then became necessary to demonstrate that these intervals, which are produced by the ratios from the first six numbers, are as sacred within the Pythagorean and Platonic framework as those ratios produced from the first four numbers (2:1, 3:2, and 4:3). Zarlino maintained that 6 is the first perfect number (being the sum of all the whole numbers of which it is a multiple: 1+2+3=1x2x3=6) and the augmented scope of 'sacred' consonances must include the previously excluded major third (5:4), minor third (6:5) and major sixth (5:3), with the minor sixth admitted contingently as the inverse of the major third,[53] despite its ratio being outside the number six (8:5).[54] Zarlino was, consequently, the first Western music theorist to mathematically define what are now referred to

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[52] The octave, the perfect fifth and the perfect fourth. Cristiano M.L. Forster, "Just Intonation," as part of *Musical Mathematics: Western Tuning Theory and Practice,* Chapter 10, Part VI (2004), pp. 573-712. URL = http://www.chrysalis-foundation.org./musical_mathematics.htm

[53] In other words, the major third and the minor sixth make up the perfect octave; for further reading on Zarlino's inclusion of the minor sixth, see Alan C. Bowen, "The Minor Sixth (8:5) in Early Greek Harmonic Science," in: *American Journal of Philology* 99 (1978), pp. 501-506.

[54] Claude V. Palisca, "Scientific Empiricism," p.102. Note also that Descartes devotes a chapter in his *Compendium musicae* to the major and minor thirds and sixths. He places the major third ahead of the fourth (which he called the 'unhappiest of all consonances') according to its degree of perfection; see René Descartes*, Compendium of Music,* trans. by Walter Robert with introduction and notes by Charles Kent, Vol. 8 of *Musicological Studies and Documents*, ed. by Armen Carapetyan,(American Institute of Musicology, 1961).

as 'minor keys' and 'major keys'.[55]

Zarlino developed rules of composition based on the premise that intervals whose ratios are derived from within the *senario* (the numbers one through six) have permanence and universality[56]; those derived outside of the *senario* are dissonant and impermanent, their sole purpose being that of ornamentation to enhance the consonances. His treatise on music theory and composition, the *Istitutioni harmoniche*, is rife with theological overtones and became the authority in contrapuntal writing for the next century and a half. This is not to say, however, that there were no reputable music theorists who criticized Zarlino.[57] During this time there was a strong movement in the music circles of Europe that moved away from a purely mathematical approach to music theory.[58] The premises upon which he bases his theory were undermined as early as the sixteenth century by a number of scientific discoveries, the most notable performed by Giovanni Battista Benedetti[59] and

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[55] Cristiano M.L Forster, *Musical Mathematics: Western Tuning Theory and Practice,* Chapter 10, Part VI, Section 41.

[56] It is important to note that all of Zarlino's musical ratios are expressions of string length ratios, as were those music theorists who influenced him, most notably the Arabic scholars Al-Farabi (d. *ca*. 950) and Avicenna (980-1037). Scientific experimentation was being done at the time of Zarlino which sought to undermine the premises of his theory, but these experiments measured ratios with weights as opposed to length of string; see Cristiano M.L Forster, *Musical Mathematics: Western Tuning Theory and Practice*, Part VI, Section 39; Dorothea Baumann, "Musical Acoustics in the Middle Ages," trans. by Barbara Haggh, in: *Early Music* 18/2 (May 1989), pp. 294-323; William Roy Bowen, "Music and Number: An Introduction to Renaissance Harmonic Science," (Unpublished Ph.D. Dissertation, University of Toronto, 1984).

[57] See Claude V. Palisca, "Theory, theorists," p. 373.

[58] Claude V. Palisca writes "Mei typifies his generation also in wishing to lead music out of the sphere of mathematics and physics into that of poetics...There was more kinship, in his view, between the expressive arts of rhetoric, poetry, oratory and music, than between music and mathematics or cosmology." See Claude V. Palisca, trans., *Girolamo Mei: Letters on Ancient and Modern Music,* p. 81.

[59] Giovanni Battista Benedetti (1530-1590) performed a number of experiments in an effort to shed light on the mechanics of the production of consonances. He discovered that consonant intervals are a result of the compressions and rarefactions of sound waves (see Footnote 61below in Chapter 3 of this thesis) in relation to lengths of string (provided the tension remains constant); see Claude V. Palisca, "Scientific Empiricism," pp. 103-110. These discoveries may have undermined the importance of Zarlino's *senario*, but they confirmed his claim that consonant intervals are a manifestation of mathematical ratios. Note also Mersenne's experiments below in this chapter.

later, Vincenzo Galilei, a former student of Zarlino.

Zarlino's weakness lies in his attributing consonance to specific numbers, and not specific ratios. Experiments in acoustics performed by the sixteenth century mathematician Benedetti revealed that the distinction between consonance and dissonance has to do with the proportions themselves, and not with the specific numbers that measure them.[60] For instance, we know that the ratio describing the consonant octave is 2:1. Therefore, if one were to pluck the string of a monochord, and then pluck one half of the length of the string, the molecules in the air that are affected by the sound waves produced by these two different lengths would be moving proportionally (2:1) with one another. The sound produced would therefore be a consonant one, as the series of compressions and rarefactions would not be clashing against each other.[61] "Therefore, since the longer part is twice the shorter, and they are both of the same tension, in the time that the longer completes one period of vibration, the shorter completes two."[62] The ratios describing

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[60] For a broad discussion of acoustic experiments performed in the Middle Ages, see Dorothea Baumann, "Musical Acoustics in the Middle Ages"; William Roy Bowen, "Music and Number."

[61] Compressions and rarefactions refer to the movement of molecules produced by the longitudinal wave of vibration (first introduced by Girolamo Fracastoro in his *De sympathia et antipathia rerum* (Venice, 1546) as *addensatio et rarefatio*: two strings of equal length stretched to the same tension are susceptible to each other's vibrations, demonstrating that the movement of air around the vibrating strings is measurable). When this longitudinal wave reaches our eardrum, we hear sound. When two longitudinal waves are occurring simultaneously, and their waves of vibration are not in proportion with one another, then one hears noise, or dissonance. At any frequency (number of vibrations per second) other than a harmonic frequency, the resulting disturbance of the medium is irregular and non-repetitive. Any object (especially a musical instrument) that vibrates in a regular and repetitive fashion, contains harmonic frequencies that are related to each other by simple whole number ratios. The major distinction between music and noise is that noise contains a hodgepodge of frequencies that have no mathematical connection, while music contains a blend of frequencies that are mathematically relational. The interval that one *hears* as consonant, therefore, is contingent upon the relationship of the longitudinal waves being mathematically commensurable.

[62] Benedetti, *Diversarum speculationum mathematicarum & physicorum liber*; as mentioned in Claude V. Palisca, "Scientific Empiricism," p. 105. Concerning iconographical evidence with regard to the monochord divisions, see, for example, Guido of Arezzo illustrating the division of the string length to Bishop Theobaldus, in his *Micrologus* which he dedicated to Bishop Theobaldus, transmitted in twelfth-century manuscript, as preserved in Vienna, Osterreichische Nationalbibliothek; see Donald J. Grout and Claude V. Palisca, *A History of Western Music* (New York: W.W. Norton & Co., 2001 is sixth edition of 1960), p. 51.

consonances understood as the 'frequency of vibrations' are not limited by Zarlino's perfect number 6.[63] In fact, if one is to strictly adhere to the Pythagorean tuning system as Zarlino does, then difficulties arise in compositions – difficulties that can only be nullified with the musicians ear.[64]

Although ignored among the intellectuals who clung to Pythagorean ideology, Benedetti's experiments sparked interest in those more empirically-minded scholars, such as Vincenzo Galilei[65]. It was already common practice for musicians to tune consonances to sound sweeter to the ear,[66] but, as Benedetti pointed out, the unequal half-tones (or semitones) became more obvious in polyphonic music.[67] Claude V. Palisca writes:

> The two tunings that had received theoretical sanction up to now, the Pythagorean...and the syntonic diatonic of Ptolemy... were both originally devised for purely melodic music, such as that of a voice singing alone, or

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[63] For instance, the minor sixth has a ratio of (8:5) which is outside of the *senario*. Even so, Zarlino hears the interval as consonant and rationalizes it into his system. See p. 46 above in this chapter.

[64] Benedetti proved that if one adheres to the syntonic diatonic tuning of Ptolemy or the Pythagorean tuning, then it is impossible to sing polyphonically without the pitch falling. If a theoretically true Pythagorean or Ptolemeic tuning of the consonances is maintained, as one ascends by fifths (multiplying by 3:2), the pitch at the point of departure will be altered once the cycle of fifths (twelve intervals of the fifth, or seven octaves) is complete. This discrepancy (the difference between twelve fifths and seven octaves) is referred to as a Pythagorean comma (see Claude V. Palisca, "Scientific Empiricism," pp. 114-115; see also, Chapter 2, Footnote 94 above). The only way the pitch can remain true is in a well-tempered system, where the octave is divided into twelve equal half-tones. This is why the natural tendency for the musician is to adhere to a well-tempered tuning.

[65] See Claude V. Palisca, *Studies in the History of Italian Music and Music Theory,* pp. 200-235.

[66] See Archytas above in Chapter 2 of this thesis.

[67] See Claude V. Palisca, "Theory, theorists," p. 373. Theorists indoctrinated in Boethius' *Fundamentals of Music* had difficulty with the half-tone. Aside from the difficulties explained in Footnote 64 above in Chapter 3 of this thesis, the division of the octave into twelve equal semitones is based on the irrational number 'root 2'. The Pythagorean whole tone (9:8) can only be divided into a lesser and a greater, but not two equal semitones. In an equal tempered scale (where the octave is divided into twelve *equal* semitones), the semitone is not a simple integer ratio, it is the twelfth root of 2 and larger intervals are multiples of the twelfth root of two. For further explication on the division of the whole tone, and the problem of irrational numbers in Greek Music Theory, see Heinrich Vogt, "Die Entdeckungsgeschichte des Irrationalen nach Plato und anderen Quellen des 4. Jahrhunderts," in: *Bibliotheca Mathematica* 3/X (1909-1910), pp. 97-155; Jan W. Herlinger, "Marchetto's Division of the Whole Tone," in: *Journal of the American Musicological Society* 34 (1981), pp. 193-216; Jan W. Herlinger, "Fractional Division of the Whole Tone," in: *Music Theory Spectrum* 3 (1981), pp. 74-83.

> in unison with other voices or instruments. When either of these tunings was used in polyphonic music, many difficulties arose because of their unequal tones or semitones, or their harsh sounding consonances on certain steps of the scale. From earliest times instrument builders and tuners compensated for these shortcomings by tempering the consonances by ear. [68]

Consequently, the problem of tuning became the subject of much debate among music theorists.[69] One of the earliest advocates of the empirical approach to music was Vincenzo Galilei. He developed a new ranking system of consonances based solely on sense experience and was convinced that equal temperament was the only solution for instrumental music.[70] Zarlino's objection to equal temperament[71] arose from his purely theoretical and rational classification of consonances, and his unwillingness to accept any consonances outside of the *senario*. In response, Galilei claims that: "among the musical intervals, those contained outside the *senario* are as natural as those within it. The third contained in the 81:64 ratio is as natural as that in the 5:4 ratio. For the seventh to be dissonant in the 9:5 ratio is as natural as for the octave to be consonant in the 2:1 ratio."[72] Galilei refused to accept that there was any 'universal *harmonia*' in nature and strongly advocated equal temperament, thereby dismissing both the Pythagorean diatonic and Ptolemy's

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[68] Claude V. Palisca, "Scientific Empiricism," p. 114; see also Footnotes 61 and 64 of Chapter 3 above in this thesis.

[69] See Nicola Vicentino (1511-*ca*.1576) who would not settle with abstract mathematical theory, and demonstrated how his theories applied to practical composition and tuning. He is famous for his *L'antica musica ridotta alla moderna prattica (Ancient Music Restored to Modern Practice)* first published in Rome in 1555; see Bill Alves, "Just Intonation system of Nicola Vicentino," in: *Journal of the Just Intonation Network* 5, no.2 (1989), pp. 8-13, URL = http://www2.hmc.edu/~alvespubs.html.

[70] Galilei proposed a uniform semitone of 18:17 for instrumental music, but maintained that voices aspired toward a juster intonation than could not be defined (hence, root 2); see Claude V. Palisca, "Theory, theorists," p. 373.

[71] See Claude V. Palisca, "Scientific Empiricism," pp. 114-115.

[72] *Discorso intorno all' opere di messer Gioseffo Zarlino da Chioggia* (Florence: G. Marescotti, 1589), pp. 92-93; see Claude Palisca, "Scientific Empiricism," p. 122.

syntonic diatonic.[73] According to Galilei, reason is not the governing factor in creating the perfect tuning system – it is the musician's ear that will *hear* whether or not an interval is consonant or dissonant. He does not, however, entirely dismiss the notion of a correlation between what one *hears* as consonance and consistent numerical ratios – after all, he was musically trained within a Boethian context – but he considers these numerical ratios as units of measurement (measuring sounding bodies) and nothing more. His argument is an illustration of music theorists' move toward an exclusively subjective approach to music, for he holds that "the musician deals in a subjective realm in which the sense has sufficient powers unaided by the reason."[74]

Like Aristoxenus (who was neglected for the most part until now as the nemesis of Boethius), Galilei's aim was to create a musical system based on data compiled from sense experience and observation. Unlike Zarlino, Galilei was not interested in trying to fit musical facts into a purely rational and theoretical system.[75] His experiments were quite revelatory. For instance, he discovered that the octave may be brought about by *three* different ratios: in accordance with string length the ratio is 2:1 (which corresponds with linear measurement); with weights attached to strings (changing the tension in the strings) the ratio is 4:1 (analogous to surface measurements

[73] Ptolemy's syntonic diatonic falls somewhere in between equal temperament and the Pythagorean tuning; see Andrew Barker, "Ptolemy on the Harmonic Division of His Predecessors," in: Andrew Barker, *Scientific Method in Ptolemy's* Harmonics (Cambridge and New York: Cambridge University Press, 2000), pp.109-131; Andrew Barker, "Reason and Perception," in: *Scientific Method in Ptolemy's* Harmonics, pp. 14-32; Jon Solomon , "A Preliminary Analysis of the Organization of Ptolemy's *Harmonics*," in: *Music Theory and Its Sources*: *Antiquity and the Middle Ages*, Vol. 1 of *Notre Dame Conferences in Medieval Studies*, ed. by André Barbera (Notre Dame, Indiana: University of Notre Dame Press, 1990), pp. 68-84.

[74] Claude V. Palisca, "Scientific Empiricism," p. 122.

[75] Galilei was highly influenced by Florentine humanist, Girolamo Mei (1519-1594). Among other things, Mei was noted for distinguishing between music as science and music as art: he felt that dissecting art in terms of its scientific and metaphysical facts was a violation of its intrinsic subjective nature. For insight into Mei's correspondence with Galilei, a good source is Claude Palisca, *Girolamo Mei: Letters on Ancient and Modern Music.*

2^2); and with volume of concave bodies like organ pipes the ratio is 8:1 (corresponding to cubic measurements 2^3).[76] He also discovered that, in testing strings of various materials, a unison was only possible if the thickness, length, material and tension were identical.[77] Alter any one of these factors and a unison cannot occur. According to Galilei, these experiments reveal the confusion suffered by Zarlino and those theorists before him[78]: the legendary 'sounding' numbers that are allegedly manifest in music have no *actual* existence – they are only significant insofar as they measure material relationships in 'sounding' bodies. They are meaningless in and of themselves. Furthermore, one cannot rely on mathematics to judge the superiority of one tuning system over another. According to Galilei, a consonant interval that manifests an acceptable ratio may sound sweet on one instrument, yet edgy on another, depending on the material and shape of the instrument.[79] The ear judges by listening, not quantifying.

Similarly, in a letter to Marin Mersenne dated January 13, 1634, Descartes writes :

> Concerning the sweetness of consonance, there are two things to be distinguished, that is, what makes them simpler and more concordant and what makes them more pleasing to the ear. Now, as for what renders them more pleasing, this depends on the places where they are employed, and there are places where even the diminished fifths and other dissonances are more pleasing than the consonances, so it is not possible to determine absolutely that one consonance is more pleasing than

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[76] As was mentioned above, Zarlino's error lies in his accepting only those ratios that are found within his *senario*. The mathematical ratios remain consistent, but they are not limited to the number 6.

[77] Galilei's experiments indirectly support Plato's argument discussed in Chapter 2 above in this thesis: we cannot acquire perfect consonance in an imperfect world.

[78] These experiments are elegantly shown in the woodcut included in Franchino Gaffurio's *Theorica musice*, Book 1, Chapter 8; as reproduced in: Gaetano Cesari, ed., Facsimile Edition of *Franchino Gaffurio: Theorica musice* (Rome, 1934); see also Franchino Gaffurio, *The Theory of Music*, trans. by Walter K. Kreyszig, p. 48 (Figure 1.8.1).

[79] Once again, this argument lends support to Plato's position – the rougher the instrument, the farther it is from sounding a pure consonant; it is the shape and material of an instrument that will determine how truly the consonant interval can be sounded. An oboe, for instance, will sound a much purer pitch than a blade of grass between one's thumbs.

another.[80]

Descartes makes a distinction here that is seminal in his position, namely between that which is 'simpler' or 'most perfect' and that which is 'most pleasing'. His ranking of consonances became the subject of an ongoing debate between himself and Mersenne.[81] Mersenne also conducted painstaking experiments, studying the acoustics of vibrating strings and organ pipes, and measuring waves and the speed of sound; he counted music among those disciplines that can be analyzed and rationally explained, and wrote about these scientific pursuits in such works as *Quaestiones celeberrimae in Genesim* (Paris,1623) and especially the *Harmonie universelle* (Paris, 1636-7).[82] As a result of these experiments (and those performed by Benedetti and Galilei) Mersenne concluded that the relative 'pleasingness' of consonances was caused by the concurrence of the returns of vibrations. As mentioned above[83], the frequencies measured in consonant intervals adhere to a consistent pattern and mathematical relationship. Mersenne's experiments in acoustics revealed that the purer an interval sounds, the simpler the numerical ratio is that governs it; the more unpleasant an interval sounds, the more complex the numerical ratio is – and the more erratic its wave pattern.

Descartes did not accept this argument's claim as a thorough enough explanation as to why

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[80] Marin Mersenne, *Correspondance, II*, as cited in Claude V. Palisca, "Scientific Empiricism," pp. 111-112. In response to Descartes, I answer that placement is crucial – for the *only* manner in which a dissonance sounds 'more pleasing than the consonances' is in its resolution to a consonant interval. A composer will utilize the jarring sound of the dissonance to make the consonance even more gratifying.

[81] See Anthony Kenny, *Descartes: Philosophical Letters.*

[82] For a good overview and bibliography of Marin Mersenne, see Albert Cohen, "Marin Mersenne" in: *The New Grove Dictionary of Music and Musicians*, 29 vols., ed. by Stanley Sadie and John Tyrrell, (London: Macmillan, 2001),Vol. 16, pp. 468-470.

[83] See Footnote 61 of Chapter 3 in this thesis.

certain intervals sound pleasing to the listener's ear while others do not. In his *Compendium musicae* (1618), written in his youth, he does recognize a hierarchy of consonances when reduced to their mathematical ratios and he remains true to Zarlino's *senario*,[84] for at the time he was immersed in an environment that was inundated with the ideals of Greek humanism. His systematic presentation of consonant and dissonant intervals, and the lengthy chapter devoted to 'the steps or musical tones'[85], reveal music's innate mathematical nature[86] – *but* his primary focus is how these intervals are *arranged* to create the most pleasing sound.[87] One may classify the intervals in a system that adheres to a scale of simplicity; yet once they are applied to the practical art of music, the most perfect interval may sound the least pleasing. He writes that dissonances borrow sweetness from neighboring consonances and that "consonances are not, as a matter of fact, so absolute that all their sweetness will be lost if one pitch of the interval is moved a very little."[88] Descartes' distinction between 'simplicity' and 'pleasingness' paved the way for a separation of the objective (*musica theorica*) and the subjective (*musica practica*) in music, and set the stage for music's eventual push into the realm of aesthetics. This new trend in music and composition focused

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[84] Charles Kent writes: "The similarity between many of Descartes' remarks and those of Zarlino in his *Le institutioni harmoniche* (1558) and the fact that Descartes, in the *Compendium*, admits having read the work indicate that Descartes was indebted to Zarlino for many of his theories"; Charles Kent in René Descartes, *Compendium of Music*, trans. by Walter Robert, p. 9.

[85] See René Descartes, " The Steps or Musical Tones," in: *Compenium of Music,* trans. by Walter Robert, pp. 28-43.

[86] The work was, after all, written specifically for Isaac Beeckman (1558-1637), a well-known mathematician. It was Beeckman who had shown Descartes a demonstration of the correspondence between the wave motions of consonances; see Footnote 61 of Chapter 3 above in this thesis.

[87] Note the chapter devoted to 'About Composition and the Modes' where he provides his reader with basic rules of composition, most of which still holds today, especially in choral writing; see René Descartes, *Compenium of Music*, trans. by Walter Robert, pp. 46-51.

[88] *Ibid.* p. 45.

primarily on *musica practica*[89], expounding the virtues of improvisation, and liberally utilizing dissonances. Consequently the reign of Boethius' *musica theorica* that had prevailed for so many centuries and kept music in its place as one of the cornerstones of the *quadrivium*, was coming to an end. Music had been redefined and was now classified as a branch of aesthetics, not mathematics. By the end of the Renaissance, subjectivism became the normative approach[90].

Especially since the romantic movement in the nineteenth century, music is understood as a means of communicating expression and emotion. Does this in any way undermine the results of experiments performed by Mersenne and others before and after him? Absolutely not. Sound must bow down to the laws of physics, and physics must bow down to the objectivity and universality of mathematics; the purest consonances sound that way because the frequencies of the vibrating strings (or organ pipes, or bells, or a blacksmith's hammers[91]) adhere to unchanging numerical ratios. Such evidence is irrefutable, and one can make the claim that: insofar as music contains consistent mathematical truths, and mathematical truths are objective and eternal, music necessarily contains

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[89] As evinced by surviving treatises, such as Franchino Gaffurio, *Musica practica* (1492); Claudio Monteverdi, *Quinto libro de madrigali* (1605); Giulio Cesare Monteverdi, *Scherzi musicali* (1607); and Girolamo Diruta, *Seconda parte del Transilvano* (1609); see Claude V. Palisca, "Theory, theorists," p. 374-376.

[90] See, for example, Johann Mattheson; Eduard Hanslick; and Peter Kivy to name a very few.

[91] Boethius relays in Book I, Chapter 10 of *The Fundamentals of Music*, trans. by Calvin Bower, the story of Pythagoras and the smithy: "In the meantime, by a kind of divine will, while passing the workshop of blacksmiths, he overheard the beating of hammers somehow emit a single consonance from differing sounds. Thus in the presence of what he had long sought, he approached the activity spellbound. Reflecting for a time, he decided that the strength of the men hammering caused the diversity of sounds, and in order to prove this more clearly, he commanded them to exchange hammers among themselves. But the property of sounds did not rest in the muscles of the men; rather, it followed the exchanged hammers. When he had observed this, he examined the weight of the hammers. There happened to be five hammers, and those which sounded together the consonance of the diapason (8ve) were found to be double in weight..." For further commentary on this myth, see Marius Schneider, "Pythagoras in der Schmiede," in: *Festgabe zum 60. Geburtstag von Willi Kahl am 18. Juli 1953*. (Cologne, 1953), pp. 126-129; on the relevance of myth as a historical phenomena, Peter G. Bietenholz writes: "Fact and fiction – all history must include both; all that is not fact, indeed, all interpretation of the facts, must ultimately fall within the reach of fabula"; Peter G. Bietenholz, *Historia and Fabula: Myths and Legends in Historical Thought from Antiquity to the Modern Age,* Vol. 59 of *Brill's Studies in Intellectual History,*ed. by A.J. Vanderjagt *et al.* (Leiden and New York: E.J. Brill, 1994), p. 1.

objective truths that are eternal. Further, the properties of music that are discussed within the context of aesthetics – its expression, its meaning, its very composition – are temporal, corruptible and subject to change. Such dynamic and ever changing properties are contingent upon the eternal and unchanging properties that are woven to create music's very fabric. The relationship between the eternal and the temporal in music, the *musica theorica* and the *musica practica*, is not a mutually exclusive one. The relationship is one of contingency – the latter upon the former.

Chapter 4

A Shift from *Musica theorica* to *Musica practica*:

Some Contemporary Arguments in Philosophy of Music

Unlike his fellow scientist Mersenne, who felt that music's ability to sway the emotions was scientifically explicable, Descartes considered music's emotive effect on the 'heart' of the listener scientifically immeasurable. Descartes' claim reflects a shift in focus from the music itself (with its eternal properties) to the subject experiencing it. This position represents the dominant approach to music today, which is why, as a general rule, the listener is at the center of most philosophical discussions of music.[1] As the focus is on the listener, or the subject *experiencing* the music, then such discussions are necessarily concerned with music's temporal, changing and subjective properties, as opposed to its eternal, unchanging and objective properties. As a general rule, music in contemporary thought is understood in terms of its expression of emotion, meaning, imagery and colour and how such expression is received by its listener, and not in terms of its mathematical constituents. Yet how can music's expressive nature – and the recipient of such expression – remain separate from its eternal constituents, and the principle of ἁρμονία? *Harmonia* comprises the very fabric of music and with it its immutable mathematical truths. One can only experience music from within a temporal framework; yet during that brief period of time, can it not be said that one is partaking in music's eternal nature?

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[1] As Robert Walker writes: "In musical perception...when we hear a consonance, we hear it for cultural reasons, not physical ones. Musical perception is, therefore, a matter of habituation and acculturation... On the one hand, music is man's creation (its laws, its values, its forms); on the other, however, it reflects certain empirically verifiable behaviors of vibrating strings, columns of air etc. The two can only be reconciled through an understanding of man's proclivity to select for his own purposes and reasons certain vibrations rather than others. His selection is based upon cultural values rather than values attached to universal properties which would tend to attract the 'thinking', logical mind"; Robert Walker, "Music Perception and the Influence of Western Musical Theory," in: *Canadian Journal of Research in Music Education* 29 (Aug. 1987), p. 54.

To answer this question, one must first explore the implications of 'emotion' and 'meaning' in music. This chapter will observe some contemporary perspectives that are concerned for the most part with the subject experiencing music. To begin, we turn our attention to two opposing positions – that of the cognitivist, and that of the emotivist, labels borrowed from Peter Kivy.[2] In his book *Music Alone*, Kivy refers to these two positions as "an ancient quarrel that runs through the philosophy of music [concerning] the relation of music to the emotive life..."[3] Both positions attempt to provide sound explanations pertaining to the so-called immeasurable component of music, namely, its ability to express emotion and meaning. The musical cognitivist will make the claim that any emotion attributed to music exists as a property of the music itself and is merely recognized by the listener. The musical emotivist will make the claim that any emotion attributed to music exists only insofar as it is aroused in the listener. Both claims raise some obvious problems.

The cognitivist holds that specific emotions are deliberately written into the very fabric of a composition, and are therefore inherent within the music itself. If this is possible, then the listener should be able to recognize a particular emotion as a *property* of the music, not because the music *arouses* that particular emotion in the listener.[4] The cognitivist will claim that since specific emotions are properties of the music itself, then emotions communicated by the composer should be undeniably recognized by the listener. It may be true that a composer can manipulate tonal-

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[2] Peter Kivy, *Music Alone: Philosophical Reflections on the Purely Musical Experience* (Ithaca, New York and London: Cornell University Press, 1990), p. 146.

[3] *Ibid.*, p. 146.

[4] The only context where the listener can recognize, without a doubt, the emotion that the composer is trying to convey is that of opera, where the listener is following a libretto while watching the story unfold both audibly and visually.

rhythmic patterns to represent certain emotions (a minor key is thought to be sad, or a fast rhythmical pattern is considered exciting), but it is farfetched to assume that the composer can determine, without a doubt, the emotional response that will be aroused within the listener.

The musical cognitivist, however, will turn attention away from the emotions aroused *within the listener*, and towards the representation of emotions *within the music itself*. The cognitivist firmly holds that there are recognizable tonal and rhythmic patterns that can be used to represent specific emotions.[5] Music's aim is "above all, [to] produce something beautiful which affects not our feelings, but the organ of pure contemplation, our *imagination*." [6] This is not to say that music does not *move* the listener – the cognitivist will make the distinction between the *specific emotions* that the composer is expressing with deliberate tonal-rhythmic patterns, and how the listener reacts, or is moved by those emotions.

To illustrate this distinction, Kivy cites Johann Mattheson's famous *Der vollkommene Capellmeister* (1739)[7]: "it is in the true nature of music that it is above all a teacher of propriety."[8]

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[5] The following is one of seven "more or less plausible" propositions regarding music and emotion offered by Francis Sparshott: "some musical pieces are typically and properly heard as actually having the quality of a named emotion (actually *being* mournful), in the sense that the name of the emotion in question will in suitable circumstances be not merely accepted but *volunteered* as truly descriptive of it"; Francis Sparshott," Music and Feeling," in: *Musical Worlds*, ed. by Philip Alperson (Philadelphia, Pennsylvania: Pennsylvania State University Press, 1998), p. 27.

[6] Eduard Hanslick, *The Beautiful In Music* (New York: Da Capo Press, 1974 is reprint of London and New York: Novello, 1891), p. 20.

[7] Johann Mattheson, *Der vollkommene Capellmeister, Das ist Kapellmeister, das ist gründliche Anzeige aller derjenigen Sachen, die einer wissen, könnrn, und vollkommen inne haben muß,der einer Kapelle mit Ehren und Nutzen vorshehen will: Zum Versuch entworffen* (Hamburg: Christian Herold, 1739), Facsimile Reprint, ed. by Margarete Reimann as Vol. 5 of *Documenta Musicologica — Erste Reihe: druckschriften-Faksimiles*, ed. by Association Internationale des Bibliothèques Musicales and Internationale Gesellschaft für Musikwissenschaft (Kassel and Basel: Bärenreiter, 1954); also in English translation, Ernest C. Harriss, *Johann Mattheson's Der vollkommene Capellmeister: A Revised Translation with Critical Commentary*, Vol. 21 of *Studies in Musicology*, ed. by George Buelow (Ann Arbor, Michigan: UMI Research Press, 1981).

[8] Ernest C. Harriss, *Johann Mattheson's Der vollkommene Capellmeister,* p. 104 (I, iii, 54) as cited in Peter Kivy, *Music Alone,* pp. 154.

Mattheson firmly holds that a piece of music can be written with specific emotional properties. Such properties have the power to move – and ultimately manipulate – the listener. In keeping with the prominence of rhetoric within the education system of eighteenth-century Germany, Mattheson espouses the virtues of music as a kind of rhetoric[9]. His compendium of practical and theoretical music (written specifically for the Lutheran church musician) devotes a great deal of attention to the methodology involved in representing specific emotions within the music itself. With the help of Mattheson's compendium, the composer can supposedly convey particular emotions, and through *recognition* of these emotions, the listener is moved to humbly expound upon the virtues of God (or be reduced to contrition). Put simply, Mattheson's compendium is an attempt to provide tools for writing powerful musical sermons.[10]

Mattheson's treatise on music draws the aforementioned distinction between the deliberate and presumably recognizable 'emotion' that is woven into a composition, and the emotion felt by the listener as she is moved by the composition.[11] As a 'musical cognitivist', Mattheson is not arguing that a composer has the ability to evoke emotion *x* in the listener; he *is* arguing that the composer has the ability to include emotion *x* as a recognizable property of the written and

[9] For a summary of the rhetorical principle in music theoretical treatises in the Middle Ages and the Renaissance, see Joseph Smits van Waesberghe, "Die Anwendung der *ars rhetorica* in den musiktheoretischen Traktaten des Mittelalters und der Renaissance," trans. by M.U. Schouten-Glass in: *Dia-pason de monibus – Ausgewählte Aufsätze von Joseph Smits van Waesberghe: Festgabe zu seinem 75. Geburtstag,* ed. by C.J. Maas and M.U. Schouten-Glass (Buren: Frits Knuf, 1976), pp. 71-90; for further reading on the link between music and grammar, see Mathias Bielitz, *Musik und Grammatik – Studien zur mittelalterlichen Musiktheorie*, ed. by Reinhold Hammerstein *et al.* (Munich and Salzburg: Musikverlag Emil Katzbichler, 1977). For further explication on music and rhetoric since Greek Antiquity, see Paul Lehmann, "Die Inst. Orat. Des Quintilianus im Mittelalter," in: *Philologus* 89 (1934), pp. 349-383.

[10] Peter Kivy, *Music Alone*, p. 154.

[11] Movie scores are a perfect example of this. The purpose of a movie score is to convey specific, recognizable emotions in order to emotionally manipulate the audience; for an example of this, listen to any score written by John Williams of *Star Wars* and *E.T.* fame.

performed score. His conviction, however, that emotions written into a musical composition have the ability to sway the listener into a life of Christian morality is unrealistic in two ways. First, there is no guarantee that through recognizing emotion *x* the listener will be moved in the appropriate way – perhaps a musical depiction of Christ being crucified will evoke fear or revulsion instead of sympathy or contrition. Second, and this applies to the cognitivist position in general, there is no weight to the claim that emotions can be 'written into' a score – a fast rhythmical pattern could be associated with anything from excitement, to giddiness, to the ridiculous. There is no guarantee that specific tonal-rhythmic patterns represent specific emotions that *every* listener will recognize. The listener brings too much of him or herself into the musical experience, and will recognize emotions according to what is familiar.

It would seem natural, therefore, to lean towards the emotivist position; the emotivist will label a piece of music 'sad' or 'inspiring' because that is the particular emotion the music elicits in the average listener. In a counterfactual manner, Harold E. Fiske, in his essay *Why Music is Not a Theory of Emotion,*[12] adopts the position that the most common problem in theories of musical aesthetics is the "music-is-about-something-idea". He does not endorse the cognitivist's notion that music is a communication-specific system; a composer cannot communicate a specific emotion in a composition no matter how inspirational the emotion was at the time the piece was being composed. Fiske writes: "It is the profusion-of-observed-responses problem that has created much of the confusion and difficulty in developing theories that work as convincing descriptions of the listener-music relationship and listener-response activity..."[13] In other words, there are potentially

[12] Harold E. Fiske, *Music and Mind: Philosophical Essays on the Cognition and Meaning of Music* (Lewiston, New York and Queenston, Ontario: The Edwin Mellen Press, 1990), pp. 105-129.

[13] *Ibid.,* p. 108.

multifarious responses to one piece of music, even though the composer may have intended only one.

Jean-Jacques Nattiez's definition of 'meaning' does well to illustrate Fiske's argument: "An object of any kind takes on meaning for an individual apprehending that object, as soon as that individual places the object in relation to areas of his lived experience – that is, in relation to a collection of other objects that belong to his or her experience of the world."[14] This definition can serve to shed light on the reasons why one particular piece of music inspires a 'profusion-of-responses': the emotion that a piece of music evokes in a particular listener is dependent on the listener's numerous experiences and the specific emotions that each experience triggers. Fiske writes that: "appearance-value is created by the listener in which an affect, based upon the listener's own life experiences, finds ground with particular musical patterns."[15] The composer, therefore, could deliberately utilize specific 'musical patterns' to express a specific emotion, but this does not necessarily incorporate a specific – and universally recognized – emotion directly into the fabric of the piece. The emotive qualities that a listener will attribute to a piece of music will depend upon the listener's vast repertoire of memories, either conscious or unconscious. Fiske concedes that it is possible for an indefinite number of listeners to respond in kind to a particular piece of music (after all, culturally we are conditioned to a certain degree, and many life experiences are obviously shared by most of us[16]); however, according to Fiske, it is impossible to make the claim that a

[14] Jean-Jacques Nattiez is discussed in more detail below in this chapter.

[15] Harold E. Fiske, *Music and Mind*, p. 129.

[16] Another proposition offer by Francis Sparshott holds that: "the affective quality of some music is such that if a competent hearer is asked to apply to it one of two contrasted mood words or feeling words...the hearer will easily be able to comply and the responses of hearers tend to be significantly in agreement...the competence required involves not only relevant musical accomplishment but familiarity with the vocabulary used and its associated cultural codes."

composer has the ability to utilize specific tonal and rhythmic patterns in order to communicate a specific emotion, and that every listener will recognize those patterns and, consequently, hear (and possibly experience) that emotion.

As far as Fiske is concerned, the musical cognitivist's claim that emotions are properties of the music is an absurd one. Yet, the emotivist position finds its failing in the assumption that all of music's emotive qualities are contingent upon the listener's experiencing *specific* emotions. A musical experience does not necessarily entail an emotional experience. One may hear specific things while partaking in a piece of music without deliberating on it, and, further, if the listener is merely caught up in the beauty of a particular piece of music and not specific emotions, it is absurd to make the claim that the experience, and the music, is devoid of emotion. Further, the emotivist is wrong to define music by means of diverse emotions aroused in such a broad range of listeners. Disagreement is inevitable.

The question remains – what is it that connects music and our emotions? There are numerous contemporary theories that attempt to provide reasons, but, as Francis Sparshott pronounces in his essay *Music and Feeling*:

> There is much to say on the general theme of emotion and music; much that is of great value has already been said...But none of it amounts to anything that could be usefully called a *theory* of the relation between music and the emotions. We do not know what such a theory should be and have no reason to seek such knowledge...granted the intimate relation between musical practice and the affective side of life, there seems no reason a priori to suppose that only one relationship should hold between musically formal structures and the active and affective lives they relate to, or that they should relate distinctively to any specific range of such phenomena, or that such relationships as obtain should be reducible to any system.[17]

Francis Sparshott, "Music and Feeling," p. 27.

[17] Francis Sparshott, "Music and Feeling," p. 23.

He observes that the objects of experience are "experienced as multifariously affectful, and music as much as any."[18] There is no denying that there are emotions that one experiences (or observes), while listening to the object that is the musical composition. However, attempting to explain in a single theory why particular emotions are aroused within particular individuals while experiencing particular pieces of music is futile, and overwhelming. One may argue that a purely empirical theory may be possible, where specific musical phrases are correlated with specific felt emotions. This can only occur, however, in mediums that are multi-sensory such as opera, theatre or film. In these cases, the listener has something visual to reinforce any correlation between specific musical themes and particular emotions. For instance, while a character is mourning lost love, the orchestra may play a discordant and disturbing musical phrase, or perhaps a sad and pitiful one, or maybe even a melody that is sweet and melancholy – whichever musical theme is sounding will be the one that the entire audience associates with that character's loss. Yet the discussion at hand is not about to branch into other such mediums, for then music would be understood only secondarily (that is, as a mere tool used to enhance a *visual* performance) and the question that asks whether or not there can be a theory explaining the correlation between music (in and of itself) and emotions will remain unanswered. We must focus on the problem at hand, which seeks an explanation regarding the multifarious emotional responses to a single piece of music.

Remaining within the temporal camp, we turn our attention to Jean-Jacques Nattiez, whose general theory of *musical semiology*[19] attempts to provide such an explanation. In his attempt to

[18] *Ibid*., p. 27.

[19] Nattiez prefers semiology over semiotics, as the former embraces a much broader range of potential meanings with its dialectical process. Semiotics, on the other hand, is a branch of linguistics that concerns itself specifically with the signifier-signified relationship of language. Such a system is stable, invariant, and scientifically explicable. Music,

extrapolate meaning from the musical experience, Nattiez determined that there can be no simple theory explaining the causal connection between the object that is the musical composition and the subject experiencing it; the musical experience is an ongoing dialectic involving creativity, creation and interpretation. Although he concedes that there is some kind of "total musical fact"[20] somewhere out there, we can only partially grasp it as it is ever-changing and continuously re-experienced.

Nattiez understands meaning in music as "constituted by an open-ended interpretive process constrained only by sounds and the lived experiences of those engaged with them...its meanings are relative to a potentially infinite range of interpretive variables."[21] In other words, it is not a simple matter of perceiving music as a form of communication with fixed signifiers that indicate particular meanings for the listener. What is occurring is more of a dynamic and perpetual dialectic. His 'tripartitional' model of referring (adopted from Jean Molino[22]) involves the poietic, the immanent and the esthesic. The poietic is the creative force that motivates the creation of an object (a piece of music for example), which is what inspires the bringing about of an idea into an empirical form. Unlike the cognitivist's claim that meaning and emotion are objective properties of a composition that reveal the composer's intentions, Nattiez holds that the object created (the immanent) can only

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according to Nattiez, does not convey meaning in so closed a system. See the introduction to Jean-Jacques Nattiez, *Music and Discourse,* pp. 3-37.

[20] A term Nattiez uses often in *Music and Discourse.*

[21] Wayne D. Bowman on Jean-Jacques Nattiez, as cited in: Wayne D. Bowman, *Philosophical Perspectives of Music* (Oxford: Oxford University Press, 1998), p. 201.

[22] Jean-Jacques Nattiez, *Music and Discourse*, p. x. From nineteen sixty-eight to nineteen-seventy, Jean-Jacques Nattiez studied semiology under Jean Molino at Aix-en-Provence. For a brief biography of Jean-Jacques Nattiez, see Jonathan Dunsby, "Jean-Jacques Nattiez," in: *New Grove Dictionary of Music and Musicians*, 29 vols., ed. by Stanley Sadie and John Tyrrell (London: Macmillan, 2001), Vol. 17, pp. 707-708.

infer the poietic process and the intention of the creator. This is where the interpretive (or esthesic) process comes in, and such a process is flexible enough to accommodate the "entire lived experience" of participants.[23] The 'total musical fact' lies at the very core of the process, but is inaccessible in and of itself.

This semiological process leaves behind the created musical object – and the creator of that musical object – early on. Any meaning or emotions that may be attached to it is left completely in the hands of the listener and an ongoing interpretive process. This "tripartitional conception of semiology as applied to 'thinking about music'"[24] draws its dialectical format from the writings of Charles Sanders Pierce (1839-1914)[25], who presents an open, infinite system of the signifier-signified-interpretant relationship in an attempt to understand the meanings that signs generate. Meaning, in such a system, is never exhausted, as there is an infinite web of interpretants for any sign. One begins with the signifier, and that which is signified. The two are mediated by an interpretant, which in turn, becomes the signifier, which is mediated by an interpretant, which in its turn becomes the signifier, and so on *ad infinitum*. Similarly, Nattiez claims that "the poietic lurks under the surface of the immanent [and] the immanent is the spring-board for the esthesic"[26], with the poietic and the immanent being mediated by the esthesic, which then in turn becomes the poietic, which is again mediated by new esthesic and so on. "Analysis never stops engineering a

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[23] Jean-Jacques Nattiez, *Music and Discourse*,. p. 166.

[24] *Ibid.* p. x.

[25] *Ibid.* pp. 5-6.

[26] *Ibid.* p. 29.

dialectical oscillation among the three dimensions of the object".[27] The material object remains neutral and Nattiez goes so far as to conclude that "the object of the sign is actually a *virtual* object, that does not exist except within and through the infinite multiplicity of interpretants, by means of which the person *using* the sign seeks to *allude to* the object."[28] There are multiple modes of potential interaction among the poietic, the immanent, and the esthesic levels, and numerous perspectives to which any given sign may refer. Focus has shifted from the musical object itself to an infinite myriad of interpretants (or esthetic processes). Music's very definition, therefore, and any meaning that music might convey, is utterly reliant upon the experience and social and cultural context of the listener, for "it is never guaranteed that the webs of interpretants will be the same for each and every person involved in the process."[29] Nattiez leaves us with an understanding of the musical *object* (or the eternal and unchanging "total musical fact") as something that is inaccessible, and that meaning and expression in music can never be explicitly pinpointed for it exists as a continuous dynamic and open-ended process with an unlimited ceiling (which, of course, remains temporal and ever changing).

The nominalistic views of Nelson Goodman would not concede to there being *any* "total musical fact". According to Goodman, there is no way the world actually 'is', only different ways the world may be understood, or made to make sense. Symbols (and any given system of symbols) have no objective reference. Even though by definition they are referential, they do not refer to any specific objective reality; they are merely cognitive tools that we employ to construct reality.

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[27] *Ibid*. p. 32.

[28] *Ibid*. p. 7.

[29] *Ibid*. p. 11.

Goodman does not place music's value in any kind of capacity that provides insight into 'ultimate' reality, sentient or transcendent. Like all symbolic activity, music is an act of 'world-making'.[30] Goodman flatly rejects notions like essences and universals and looks upon human 'reality' as a human construction. Music's primary function, therefore, is not its ability to transcend the listener, or reveal any 'truths'. Its primary function, like all symbol systems, is reality construction.

Wayne D. Bowman writes of Goodman, that he: "rejects the idea of one largely undifferentiated, generic symbolic process in favour of an intricate and highly nuanced system of symbolic modes: each distinctive in logical structure, and each making its unique contribution to the essential symbolic mission of creating reality, of making worlds."[31] Symbols are most often thought of as a representation of 'reality', and consequently representation is considered to be almost a physical process like mirroring. Goodman's understanding of representation moves away from such "perverted ideas"[32]; symbol systems according to Goodman, could never 'mirror' reality. As mentioned above, there can only be a system of symbolic modes, for each individual will create a reality that is unique to themselves, and dependent on previous experience and its socio-cultural context. He therefore moves toward a recognition of representation as "a symbolic relationship that is relative and variable."[33]

When one seeks to copy or imitate, such representation will always involve bias – the eye will recreate in accordance with its own perception of the world and whatever experiences and

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[30] See Nelson Goodman, *Ways of Worldmaking* (Indianapolis, Indiana: Hackett Publishing, 1978).

[31] Wayne D. Bowman, *Philosophical Perspectives of Music,* p. 225.

[32].Nelson Goodman, *Languages of Art: An Approach to a Theory of Symbols* (Indianapolis, Indiana: Hackett Publishing, 1976), p. 43.

[33] *Ibid.,* p. 43.

needs that have molded that world. Goodman holds that the eye "selects, rejects, organizes, discriminates, associates, classifies, analyzes, constructs"[34], and that if a symbol was merely a reflection of some kind of objective reality, there would be no room for the unique rendering of the world that each individual has. Consequently, there would be no room for the creative process at all. A symbol, according to Goodman, not only *does* not, but *can* not, mirror: it creates.

Such a claim is bound to run into some difficulties, for language, as a symbol system, could not be continuously recreating an individual's reality when its sole purpose is communication with other individuals. In keeping with this thesis, however, let us consider works of art. It is necessary for the artist to utilize symbols that *are* mirrors of an objective reality, in order to incorporate – or *recreate* – those symbols into a work of art. An individual partaking in the experience of a 'work of art' will inevitably interpret the recreation of familiar symbols according to her own experiences, but that is another matter. The fact remains that if symbols were creations unique to each individual, there would be no point of reference with which to communicate. Despite the fact that objects of art are, generally speaking, a creative interpretation of the world around us, and regardless of the medium from which they emerge, they are still (for the most part) universally *recognized* as works of art.

Goodman admits that music manifests a much more complicated symbol system, with a far more complex network of reference (for that is what symbols do, they *refer*). According to Goodman, musical language is unique in that it does not denote any fixed meaning or emotion. Any meaning or emotion that is attached to a piece of music belongs solely to the individual experiencing the music as part of their own 'world-making'. Goodman is not concerned at this point

[34] *Ibid.,* pp. 7-8.

with musical notation (where a written symbol represents a particular pitch, or a collection of pitches as in the case of ligature), but with the immediacy of the musical experience and the emotions and meaning derived from that experience. Music's ability to engage the listener (and the immediacy with which it does so) has to do with music's *possessing* that to which it refers, instead of *pointing toward* that to which it refers.[35] While we are engaged in a *musical* experience, we are simultaneously engaged in an *emotional* experience associated with the music; the musical experience, therefore, becomes part of our world-making.

Goodman does not see music as a symbol system mirroring any kind of 'eternal reality', he sees it as a creative tool utilized in reality construction. As in the example of works of art offered above, the problem with Goodman's position with regard to music lies in his claim that symbols can only create, and that they do not point to, or mirror, *any* kind of objective reality. In order to create a piece of music, the composer must utilize symbols that *are* mirrors of an objective reality in order to recreate something unique. Music is a manifestation of existing numerical ratios, and when utilized and manipulated according to the composers own intentions (and experiences), the end result is something that is universally *recognized* as music, but experienced in a multitude of ways.

This chapter has offered four general positions thus far: the cognitivist (Johann Mattheson); the emotivist (Harold E. Fiske); the 'tripartitional' dialectical (Jean-Jacques Nattiez) and the nominalist (Nelson Goodman). All four positions attempt to understand how and why emotion and meaning are forever bound to the musical experience. The claims offered by each position focus primarily on the subject experiencing music, hence, the temporal. Discussions involving meaning and expression in music necessarily focus primarily upon the listener; each listener will approach

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[35] See Nelson Goodman, *Languages of Art*, pp. 45-95.

a piece of music from within the context of their ‘world’, a world which they have built from within a social and cultural context, and through experiences, expectations and desires. All four positions are woven with strands of truth: the listener experiences emotion *x* while listening to a musical composition, the listener strives to extrapolate meaning *x* from music, the musical experience *is* a living dynamic and ever-changing phenomenon that is forever undergoing reinterpretation, and the music symbols that exist within the musical language *have been created by us* in order that we may share the musical experience. The musical experience, however, is not contained within the temporal framework of the subject. What is occurring during a musical experience transcends the temporal, for the subject experiencing a piece of music is partaking in the eternal principles that serve as music’s most fundamental constituents. To make the claim (as Goodman does) that there is no essential and eternal musical fact is ludicrous, for it is the *musical fact* that the musical experience is contingent upon.

If one is to assert that all musical experience as we understand it is contingent upon an independently existing and eternal musical fact, it is important to clarify what is meant by this claim. In order to shed some light, it will be said of this musical fact that it is, in an Aristotelian sense, a kind of ‘cause’.

Causality is relational: if *x* provides the conditions to bring about *y*, then *x* is thought to be the cause of *y* and *y* is thought to be the effect brought about by *x*: *y* is therefore contingent upon *x*. However we create and recreate music, or whatever attributes we ascribe or interpretations we extrapolate, such endeavors are subject to change and difference of opinion. Insofar as we *experience* music, it exists contingently[36]. In a temporal sense, the musical experience is ephemeral.

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[36] To exist ‘contingently’ is to be subject to corruption and cessation.

On the other hand, music understood *in and of itself* – music *qua* music – is the manifestation of the immutable principle of ἁρμονία, its very essence woven with incorruptible mathematical truths that exist necessarily.[37] But the following question has yet to be answered: in what way is music *qua* music a cause?

There are different kinds of causal relationships, but those offered by Aristotle (in his attempt to provide a general explanation of the 'why' of the world) best illustrate the manner in which music *qua* music is a cause. The four causes of Aristotle are as follows:

Material Cause:	In one way, then, that out of which a thing comes to be and which persists, is called a cause, e.g. the bronze of the statue, the silver of the bowl, and the general of which the bronze and the silver are species.
Efficient Cause:	In another way, the form or the archetype, i.e. the definition of the essence, and its genera, are called causes (e.g. of the octave the relation of 2:1, and generally number), and the parts in the definition.
Formal Cause:	Again, the primary source of the change or rest; e.g. the man who deliberated is a cause, the father is cause of the child, and generally what makes of what is made and what changes of what is changed.
Final Cause:	Again, in the sense of end or that for the sake of which a thing is done, e.g. health is the cause of walking about... The same is true also of all the intermediate steps which are brought about through the action of something else as means towards the end...all these things are for the sake of the end, though they differ from one another.[38]

If one is to presuppose that music has inherent eternal principles, and as such music *qua* music is a kind of underlying cause of all musical experience, then, which of the four causes best describes

[37] To exist 'necessarily' is to be incorruptible.

[38] *Physics*, 194b16-195a4, *The Complete Works of Aristotle,* Vol. 1, ed. by Jonathan Barnes.

it? I propose that it lays claim to two of the four: efficient and formal.

To illustrate this proposition, let us consider a musical experience.[39] The material cause is, of course, the written or improvised composition, and the final cause is the purpose for which the musical experience was brought about in the first place, which stems from the intentionality of the composer and/or performers, and the socio-cultural umbrella under which the experience comes into being. The material and final causes are secondary and accidental – they encompass expression, colour, meaning, and all of the affectations that makes each piece of music distinct[40]. The efficient cause is the creative element that is necessary for any music to come into being[41], and the formal cause is the underlying structure that allows one to recognize it as *music*[42]. The efficient and formal causes are primary and eternal – they encompass the immutable mathematical truths that comprise the very fabric of the musical experience, and the *harmonia* that bounds them together.

There are two potential problems that inevitably surface at this point, and were alluded to by Goodman in particular. One problem is that of cultural diversity, especially with regard to music's structure and the total musical experience. A second problem is this: if music can be reduced to a complex system of symbols that is a necessary part of our world-making, then positing an eternal musical fact (or cause) is pointless. There is no need for it.

It cannot be denied that music is structurally diverse, depending on the cultures and the sub-

[39] This includes any musical experience, cross-culturally.

[40] Goodman's argument rests solely in *material* and *final* causes.

[41] See Nattiez's classification of the "poietic" *Music and Discourse*, pp. 15-17.

[42] Nattiez claims that "the border between music and noise is always culturally defined – which implies that, even within a single society, this border does not always pass through the same place; in short, there is rarely a consensus"; Jean-Jacques Nattiez, *Music and Discourse,* p. 48. I beg to differ on this point, for there is a general understanding, cross-culturally, as to what defines an experience as a musical one. Labeling a series of unorganized, disturbing and unpleasant dissonances 'music' is not making music, it is merely making a solipsistic statement.

cultures that provide the context for its creation. The musical experience changes from person to person, region to region and country to country. However, when considering the very bare bones of musical structure, the basic building blocks of music, such as the perfect octave, the perfect fifth, the perfect fourth, *and* the pentatonic scale[43] remain untouched. How such building blocks are *utilized*, however, will differ from culture to culture. Two very diverse forms of music that emerged from two autonomous cultures – Balinese and North American Aboriginal – demonstrate this fact beautifully.

If one were to compare Balinese music with Western music, one would assume that they emerged from completely different musical principles. This is not the case. Upon closer inspection, one would discover that the *slendro* scale in Balinese *gamelan* music and the pentatonic scale in Western music are, in fact, structurally the same: both are divided into five equal parts within the parameters of the octave, the most fundamental consonant interval of all (2:1). Further, the Indonesian *pelog* is constructed in a similar fashion to the diatonic scale, but with wider fourths to create a shimmering sound that occurs as a result of the rapid beating between upper harmonics.[44] This 'shimmer' sounds strange to Western ears because it developed within the context of Balinese culture; upon closer inspection, however, it is clear that the *pelog* stems from the same roots as Western music. In other words, the basic principles that underlie the two diverse musical experiences are the same – the manner in which such principles are manipulated will ultimately

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[43] The pentatonic scale can be described as a diatonic scale that lacks semi-tones C D E G A C. For further reading on the pentatonic scale in Balinese music, see Benjamin Brinner, "A Musical Time Capsule From Java," in: *Journal of the American Musicological Society* 46/2 (Summer 1993), pp. 221-260 (especially pp. 228-229); Natalie Kuzmich, "Making Connections: The Sound of Different Traditions – Part 2," in: *Canadian Music Educator* 44/2 (Winter 2002), pp. 13-17.

[44] For further reading on Balinese music, see Neil Sorrell, *A Guide to the Gamelan* (London: Faber and Faber, 1990).

depend upon the socio-cultural context (that is, the material and final causes).

Similarly, the North American aboriginal cultures (particularly the Inuit) have a very unique and distinct musical heritage that is seemingly unlike any other. Robert Walker writes that "analysis of North American Indian melody has revealed no common use of 'perfect' intervals, no theory based upon acoustic ratios and no proclivity to use certain intervals above any other. Quite random melodic movements appear to form the basis of their melodies." Yet he then goes on to say that "of some importance is the fact that these '*random*' melodies are easily remembered by singers and are passed down to later generations intact."[45] Musical patterns that are memorable enough to be 'easily remembered' are not random. Further, these 'random' melodies are recognizable *as melodies* by those who have *not* been raised within the boundaries of these cultures. The manipulation of basic core intervals *is the same.*[46] The strategic arrangement of dissonance and consonance (especially in throat-singing[47]) *is the same*. If it were *not* the same, music of the North American aboriginals (along with all aboriginal peoples around the globe) would be completely and irreversibly *unrecognizable* as a 'musical experience' to those outside of their cultural sphere. This, however, is not the case.

If Goodman's is correct in his claim that musical symbols are merely a component of our

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[45] Robert Walker, *Music Perception,* pp. 54-55.

[46] The pentatonic scale is commonly used in Inuit music. See Elaine Keiller, "Amerindian Music: Canada: Arctic Region," in: *The New Grove Dictionary of Music and Musicians,* 29 vols., ed. by Stanley Sadie and John Tyrrell, (London: Macmillan, 2001), Vol. 1, p. 492.

[47] Throat-singing creates harmonic resonances with the mouth, throat and lips. This unique method of vocalization can create up to six simultaneous pitches by one singer. The North American Inuit will have two throat singers manipulating sound with their own mouth and that of their sparring partner. For a more in depth look at throat-singing, see Elaine Keiller, "Amerindian Music," pp. 492-493; Jean-Jacques Nattiez, "The Rekkukara of the Ainu and the Katajjaq of the Inuit: A Comparison," in: *Le monde de la musique* 25/2 (1983), pp. 33-42; Jean-Jacques Nattiez, "Some Aspects of Inuit Vocal Games," in: *Ethnic Music*, 37 (1983), pp. 457-476.

'world-building' (material and final cause) and that the 'universal language' of music, and musical symbols can be reduced to creative tools that we use to construct reality, then why is it that the most basic constituents of music are the same in every culture? How is it that these same fundamental 'tools' are used in every known system of music in every known place in our world? Individuals from completely diverse cultural contexts can share in a musical experience and recognize it as such. The response to the musical experience may differ depending upon the individual, but a response occurs nonetheless – all are moved in one way or another. And every individual understands that they are somehow *involved* in something musical.

To be involved in a musical experience is to partake in its essence – an essence that is immutable, universal and unchanging irrespective of its changing corporeal nature. It can be argued that there is something more to music than mere arithmetic, and that this 'something more' encompasses the entire 'lived' musical experience. However, this 'lived' experience, this on-going dynamic dialectic, this complex interplay of symbols, exists entirely within our temporal sphere and is contingent upon the eternal mathematical laws governing the core constituents of music *qua* music. These eternal constituents of music are what we draw upon to manipulate and create; and as we experience music we partake, however briefly, in the eternal principle of *harmonia* that manifests itself in the very fundamental constituents of our universe, from the infinite depths of galaxies to the smallest component of the atom. Such is music, in and of itself.

Chapter 5

Conclusion:

Redefining the Interdisciplinary Nature of Philosophy of Music

When considering a "philosophy of music", the distinction that differentiates *musica theorica* and *musica practica* is an important one. The purpose of this distinction, however, is not to place *musica theorica* and *musica practica* into a dichotomic context, but to elucidate the contingent and interrelated relationship that exists between these two approaches; for it has been made evident throughout this thesis that, despite the turnaround that occurred in musical scholarship with regard to the pre-eminence of either *musica theorica* or *musica practica* from Greek Antiquity through to present day, there has *always* existed an underlying tension between the two.

The Pythagoreans and non-Pythagoreans of Greek Antiquity understood music, first and foremost, in terms of its mathematical nature. Music scholars and philosophers alike, recognized the consistency of music's inherent arithmetic ratios and identified these ratios with the most fundamental constituents of the cosmos. We have seen the influence that Pythagoras and his followers had on musical scholarship for at least two millennia.[1] Yet, despite the pre-eminence of *musica theorica*, there was still the musician's ear to contend with, and subsequently those arguments that placed *musica practica* ahead of *musica theorica*. If we recall, Archytas made *attempts* at reconciling pure mathematical theory with the musician's ear, but there were discrepancies in his tuning systems, and in the end he kept true to Pythagorean doctrine and never

[1] For further explication on this point, see Frieder Zaminer, "Pythagoras und die Anfänge des musiktheoretischen Denkens bei den Griechen," in: *Jahrbuch des Staatlichen Instituts für Musikforschung Preußischer Kulturbesitz* 1979/1980, pp. 203-211.

questioned the infallibility of the inherent arithmetical ratios.[2] Aristoxenus, on the other hand, disregarded the purely mathematical approach, and argued that a science of harmonics must begin with the practicing musician who *hears* whether or not an interval is consonant or dissonant without knowing anything of its inherent numerical ratio.[3] One can see, therefore, that as early as the fourth century B.C.E., at a time when the discipline of music was understood as a rigorous science, there existed a tension between the purely mathematical approach and the strictly empirical approach to the science of harmonics. This undercurrent of tension continued unabated throughout the Middle Ages and intensified even further in the Renaissance.[4]

From the sixth century C.E. until the Renaissance, *musica theorica* remained in the foreground with Boethius' seminal treatise, the *De institutione musica*. Considered the primary authoritative source for music scholars until the sixteenth century, it could very well have been intended specifically for the *quadrivium* as a pedagogical tool alongside his *De institutione arithmetica*. The objective of the *De institutione musica* is to incite its reader into a contemplative approach to music; *musica theorica* is placed ahead of *musica practica,* and an entire book is devoted to the existing tension between *musica theorica* and *musica practica.* Eventually, toward the end of the sixteenth century, music's place as one of the four scientific disciplines of the *quadrivium* underwent much scrutiny and the tension between *musica theorica* and *musica practica*

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[2] See Chapter 2 above in this thesis.

[3] See Chapter 2 above in this thesis.

[4] Note that, although musical scholars such as Franchino Gaffurio devoted separate volumes to *musica theorica* (*Theorica musice* (1492)) and *musica practica* (*Practica musicae* (1496)), *musica theorica* was still thought to have pre-eminence over *musica practica*; see Franchino Gaffurio, *The Theory of Music*, trans. by Walter Kreyszig.

reached a breaking point.[5] Instead of resolving this tension, the trend in musical scholarship sought to eradicate it entirely, merging the two disciplines with a decided emphasis on *musica practica.*

From this pivotal point in the history of musical scholarship to present day, philosophy of music has kept itself within the discipline of *musica practica*, devoting itself to the subject partaking in the musical experience. It is taken for granted, therefore, that philosophical discussions about music must be subjective in nature. *Musica theorica*, the discipline that seeks objective truths about music, has been thoughtlessly neglected and consequently "music theory" has been redefined as a subsidiary component to musical performance – understood as a given set of rules and nothing more. The extraordinary significance of the fundamental mathematical principles underlying these rules is no longer dwelled upon, and therefore the most astounding property of music is overlooked.

If we are to redefine the interdisciplinary nature of philosophy of music, we must redefine music itself. We must revisit those scholars who understood the ontological significance of music's inherent mathematical principles. Not only did they recognize the need for an understanding of music that encompasses both *musica theorica* and *musica practica*, but they also apprehended music's eternal nature, and, therefore, the necessary contingency of *musica practica* (the temporal and ephemeral component of music that we understand as the "musical experience") upon *musica theorica* (the eternal component that expresses itself through unchanging numerical ratios). It is a wonder that the pre-eminence of *musica theorica* over *musica practica* has been forgotten in philosophy of music, for throughout Greek Antiquity and the Middle Ages, and well into the Renaissance, music was understood as being rooted in the intellect, with its harmonious branches spreading from those roots and into the musical experience. Guido of Arezzo poignantly illustrated

[5] Claude V. Palisca, "Scientific Empiricism," p. 92.

this concept when he sardonically pronounced that "great is the difference between musicians and singers: the latter [merely] sing, the former know what music is made of. One who does what he does not know is defined to be an animal."[6]

[6] Guido of Arezzo, *Regulae rhythmicae* as found in *Musicae Guidonis regulae rhythmicae,* ed. by Martin Gerbert, pp.25-34. Vol. 2 of *Scriptores ecclesiastici de musica sacra potissimum ex variis Italiae, Galliae et Germaniae codicibus manuscriptis collecti et nunc primum publica luce donati.* 1784, rpt. (Milan: Bollettino Bibliografico Musicale, 1931is reprint of 1784); *Guidonis Aretini tres tractutuli editi cum apparatu critico: Regulae dictae Rhymicae,* Vol. 4 of *Divitiae Musicae Artis (Series A).* (Buren: Frits Knuf, 1985). For the English translation, see Franchino Gaffurio, *The Theory of Music*, trans. by Walter Kreyszig, p. 43. This quote is Guido of Arezzo's acrimonious articulation of the distinction between the *cantor* and *musicus* that was made by Isadore of Seville (559-636), a prominent encyclopedist of the Middle Ages. For further reading on this distinction, see Erich Reimer, "Musicus – Cantor," in: *Handwörterbuch der musikalischen Terminologie*, ed. by Hans-Heinrich Eggebrecht (Wiesbaden: Franz Steiner, 1978), pp. 1-13; Erich Reimer, "Musicus und Cantor: Zur Sozialgeschichte eines musikalischen Lehrstücks," in: *Archiv für Musikwissenschaft* 35 (1978), pp. 1-32.

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